Statistical modeling and analysis of neural data (NEU 560), Fall 2020

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Lecture 5

des these slides: brief warmup on linear systems
Linearity / Linear Systems

- a *linear system* is a mapping $f(\vec{x}) \rightarrow \vec{y}$ that has the following two properties:

1. homogeneity ("scalar multiplication"):
   $f(ax) = af(x)$ for any scalar $a$

2. additivity:
   $f(\vec{x}_1 + \vec{x}_2) = f(\vec{x}_1) + f(\vec{x}_2)$
Linearity / Linear Systems

Equivalent definition:

• A linear system is a mapping \( f(\vec{x}) \longrightarrow \vec{y} \) that obeys linear superposition:

\[
f(a\vec{x}_1 + b\vec{x}_2) = af(\vec{x}_1) + bf(\vec{x}_2)
\]

for any scalars \( a \) and \( b \).
Linear system is a kind of mapping \( f(\vec{x}) \rightarrow \vec{y} \) that obeys \textit{linear superposition}:

\[
f(a\vec{x}_1 + b\vec{x}_2) = af(\vec{x}_1) + bf(\vec{x}_2)
\]

for any scalars \( a \) and \( b \)

\textbf{It turns out:} any linear function \( f(x) \) can be written as a matrix multiplication (for some matrix \( A \))

\[
f(\vec{x}) = A\vec{x}
\]
Linearity / Linear Systems

test question:

is the function \( f(x) = ax + b \) a linear function?
Linearity / Linear Systems

test question:

is the function \( f(x) = ax + b \) a linear function?

No: it’s called an *affine* function