Perception:
The Bayesian Approach

(Discussed in chapter 6)

Lecture 18

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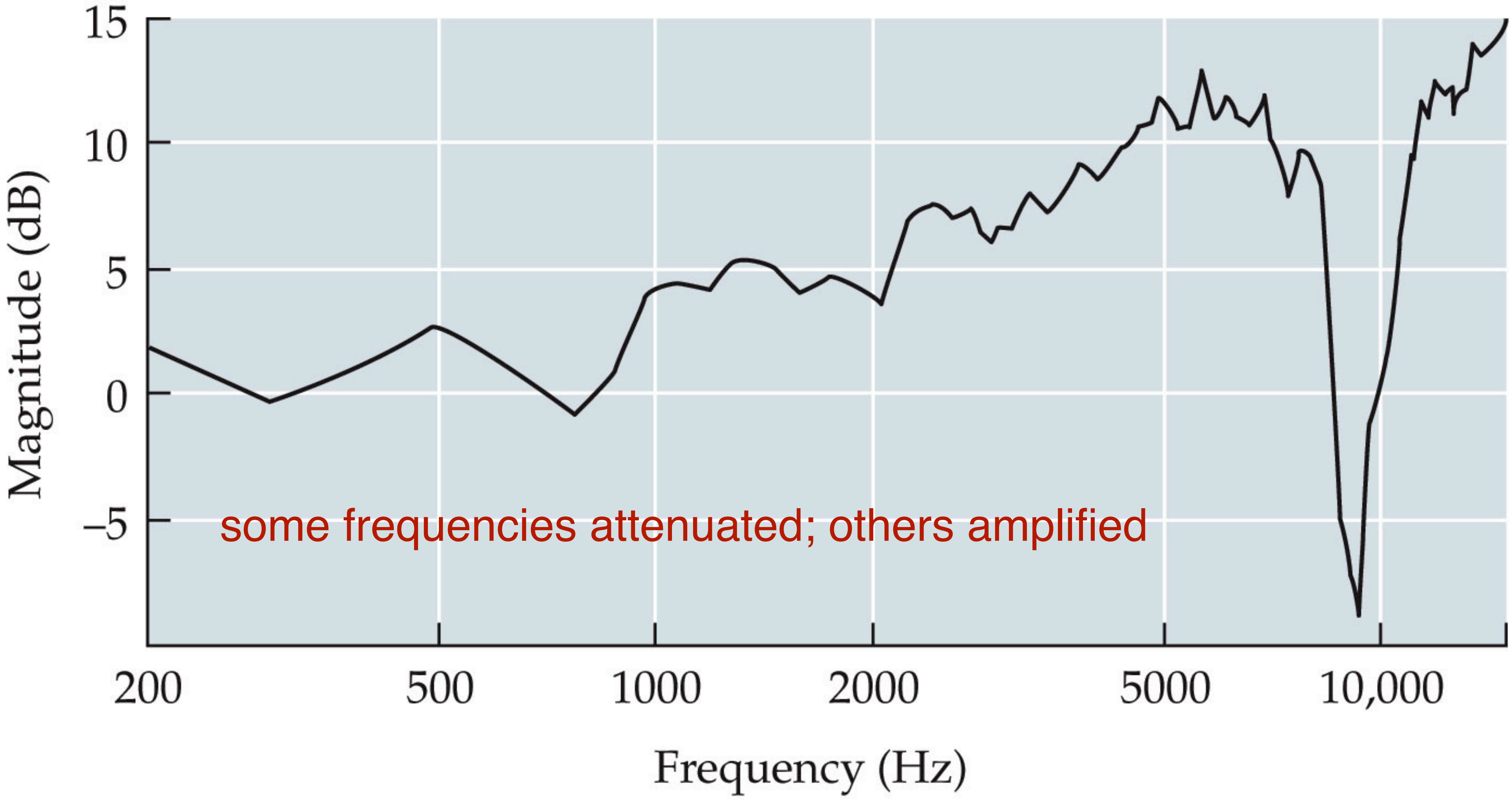
HRTF: can be measured with microphone in ear canal

HRTF for one sound source location

(30° to left, 12° above horizontal)



Head-related transfer function (HRTF)

- Hofman et al 1998: inserted plastic molds into pinnae, altering subjects' HRTFs
- sound localization performance abruptly degraded

Findings:

- Can learn a new HRTF in about 6 weeks (shown experimentally using inserted artificial pinna)
- Old HRTF is stored (can return to old one instantaneously)

Auditory distance perception

<u>Several Cues:</u>

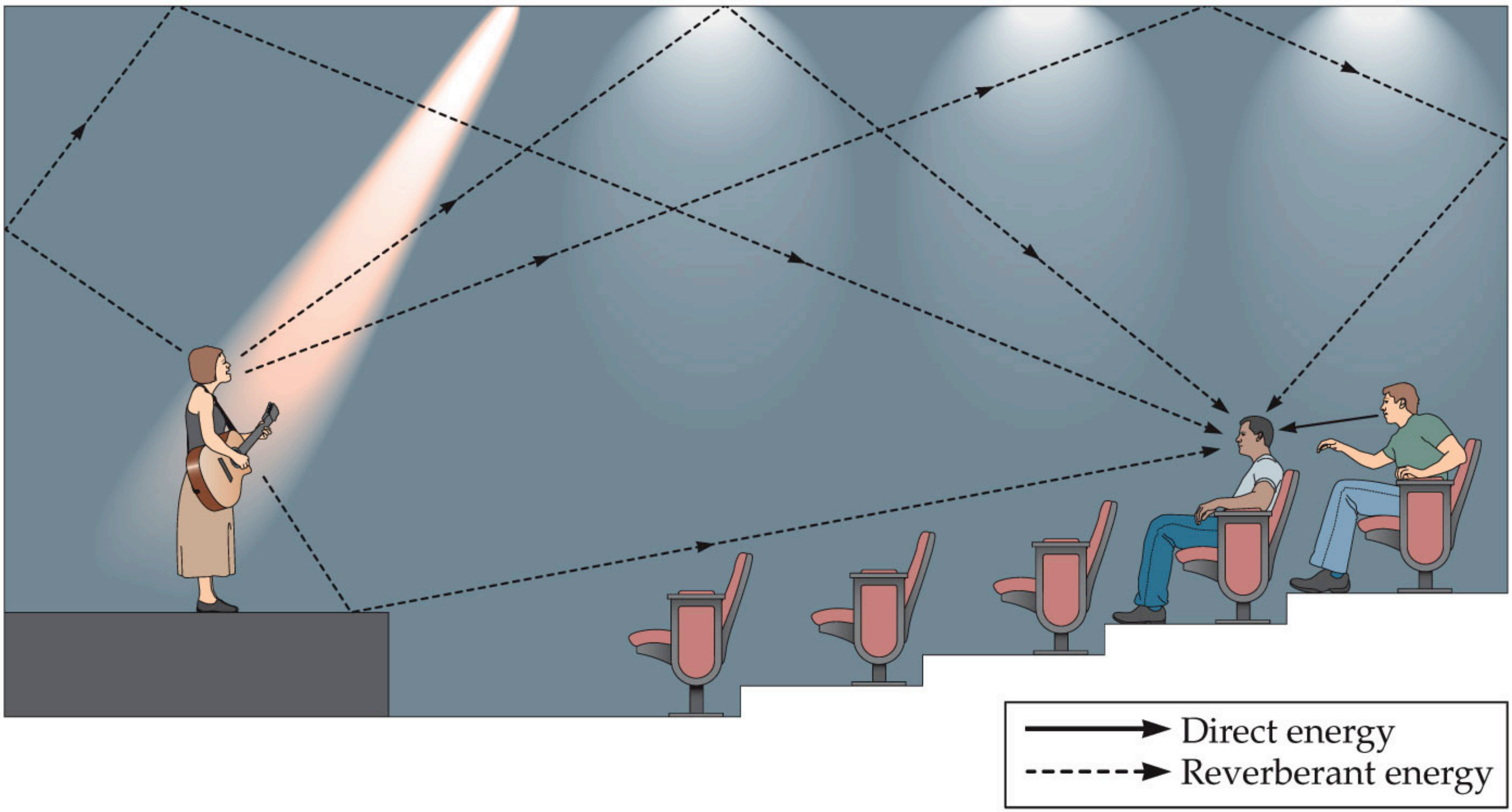
- **Loudness** (“inverse square law”) - Intensity decreases as square of the distance: (quieter = farther away) (duh.)

- **Spectral composition** - Higher frequencies decrease in energy more than lower frequencies as sound waves travel

  Example: distant vs. nearby thunder.

  - This cue only works for long distances (d > 1000m)

- **Reverberant energy** - whether most sound is arriving directly (nearby sound source) or from reverberations (far away sound source); conveyed by timing information



Auditory properties of complex sounds

(to be posted online)

Bayesian theories of perception: dealing with probabilities

Quick math quiz:

$$x + 3 = 8$$

What is x?

Quick math quiz:

$$x \times 2 = 10$$

What is x?

Quick math quiz:

$$x + y = 9$$

What are x and y?

This is an example of an *ill-posed* problem

- problem that has no unique solution

Perception is also an ill-posed problem!

Example #1:

Spectrum of Illuminant × Reflectance function of surface = Light Hitting Eye

↑

Question we want to answer: what are the surface properties (i.e., color) of the surface?

Equivalently: $X \times Y = R$ (cone responses)

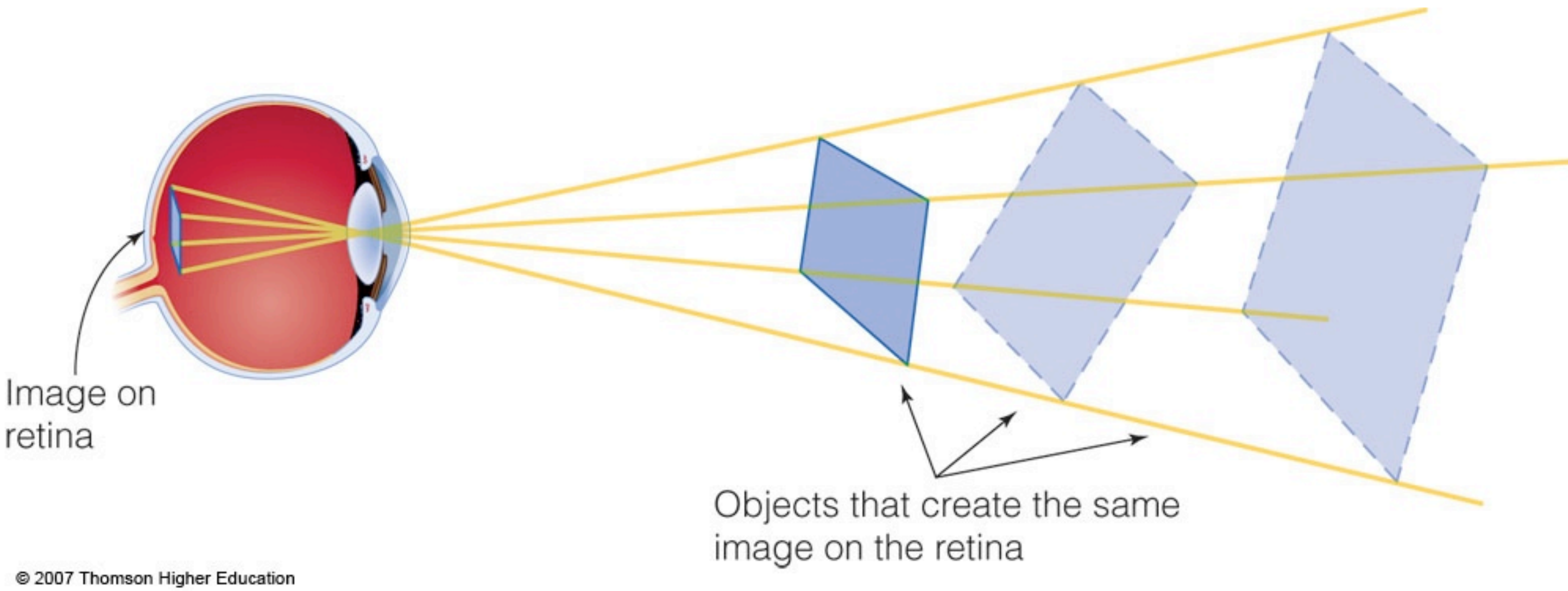
Given R, was Y?

(you'd have to know X to make it well-posed)

Perception is also an ill-posed problem!

Example #2: 3D world ⟹ 2D retinal image

Question: what's out there in the 3D world?



- ill-posed because there are infinitely many 3D objects that give rise to the same 2D retinal image
- need some additional info to make it a *well-posed* problem

Luckily, having some probabilistic information can help:

$$x + y = 9$$

Table showing past values of y:

y

7	7	7
7	7	7
5	7	7
7	6	7
7	7	7
8	7	8
7	7	7
7	7	7

Given this information, what would you guess to be the values of x?

How confident are you in your answers?

A little math: **Bayes' rule**

- very simple formula for manipulating probabilities

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

conditional probability
"probability of B given that A occurred"

probability of B

probability of A

simplified form: $$P(B \mid A) \propto P(A \mid B)\, P(B)$$

A little math: **Bayes' rule**

$$P(B \mid A) \propto P(A \mid B)\, P(B)$$

Example: 2 coins

- one coin is fake: "heads" on both sides (H / H)
- one coin is standard: (H / T)

You grab one of the coins at random and flip it. It comes up "heads". What is the probability that you're holding the fake?

$$p(\text{Fake} \mid H) \propto p(H \mid \text{Fake})\, p(\text{Fake})$$

$$(1) \quad (½) \quad = ½ \qquad \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}} = \frac{2}{3}$$

$$p(\text{Nrml} \mid H) \propto p(H \mid \text{Nrml})\, p(\text{Nrml})$$

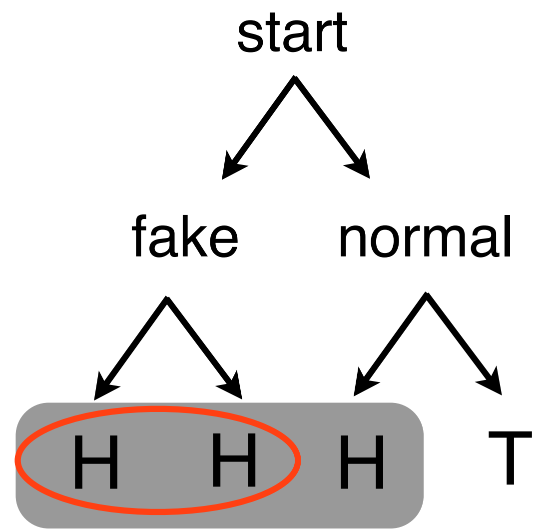
$$(½) \quad (½) \quad = ¼ \qquad \frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{3}$$

probabilities must sum to 1

A little math: **Bayes' rule**

$$P(B \mid A) \propto P(A \mid B)\, P(B)$$

Example: 2 coins



$$p(\text{Fake} \mid H) \propto p(H \mid \text{Fake})\, p(\text{Fake})$$

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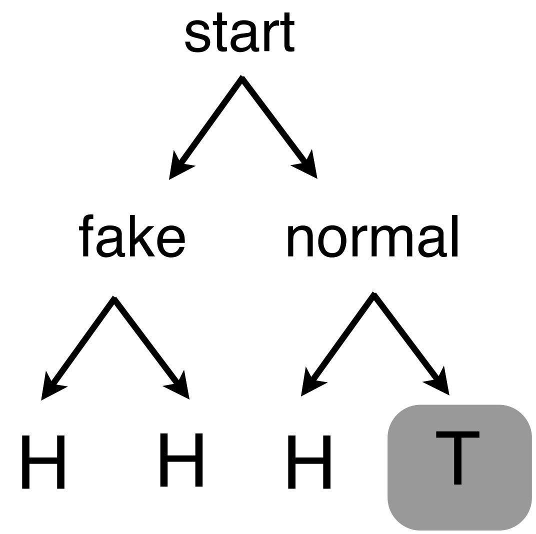
Example: 2 coins



Experiment #2: It comes up "tails".
What is the probability that you're holding the fake?

$$p(\text{Fake} \mid T) \propto p(T \mid \text{Fake})\, p(\text{Fake})$$

$$(0) \quad (\tfrac{1}{2}) \quad = 0$$

$$= 0$$

$$p(\text{Nrml} \mid T) \propto p(T \mid \text{Nrml})\, p(\text{Nrml})$$

$$(\tfrac{1}{2}) \quad (\tfrac{1}{2}) \quad = \tfrac{1}{4}$$

probabilities must sum to 1

$$= 1$$

What does this have to do with perception?

Bayes' rule: $P(B \mid A) \propto P(A \mid B)\, P(B)$

Formula for computing: P(what's in the world | sensory data)

(This is what our brain wants to know!)

B   A

$P(\text{world} \mid \text{sense data}) \propto P(\text{sense data} \mid \text{world})\, P(\text{world})$

Posterior

(resulting beliefs about the world)

Likelihood

(given by laws of physics; ambiguous because many world states could give rise to same sense data)

Prior

(given by past experience)

Helmholtz: perception as "optimal inference"

helmholtz 1821-1894

"Perception is our best guess as to what is in the world, given our current sensory evidence and our prior experience."

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"Perception is our **best guess** as to what is in the world, given our current **sensory evidence** and our **prior experience**."

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Posterior

(resulting beliefs about the world)

Likelihood

(given by laws of physics; ambiguous because many world states could give rise to same sense data)

Prior

(given by past experience)

what is perception?

prior ("top down")

statistical knowledge
about the structure
of the world

- seeing
- hearing
- touching
- smelling
- tasting
- orienting

percept

posterior

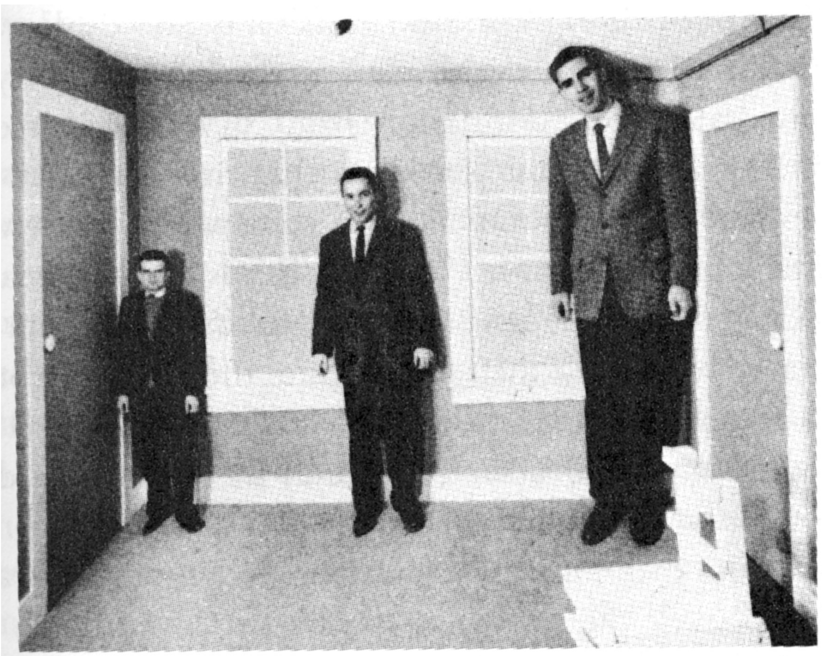
likelihood ("bottom up")

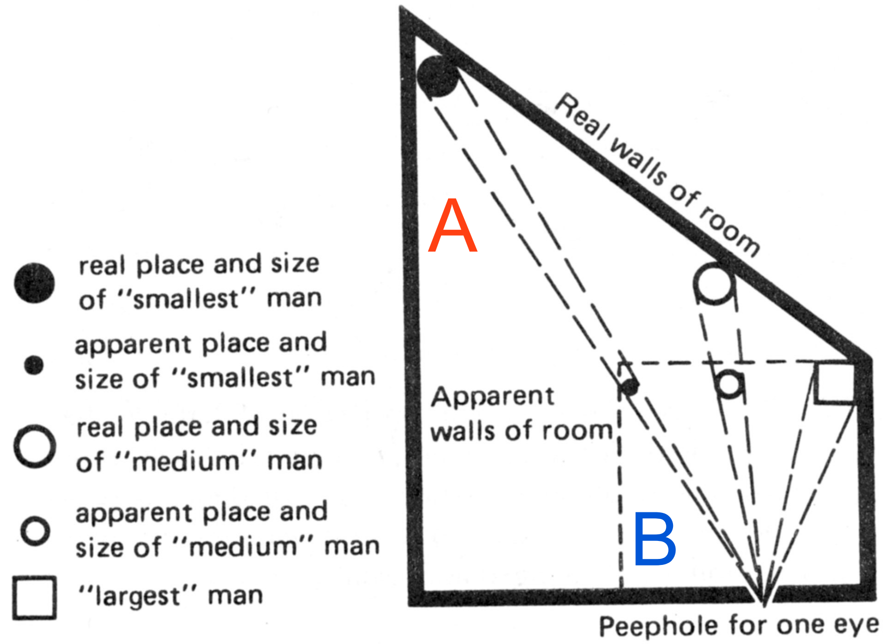
Examples

Using Bayes' rule to understand how the brain resolves ambiguous stimuli

Many different 3D scenes can give rise to the same 2D retinal image

The Ames Room





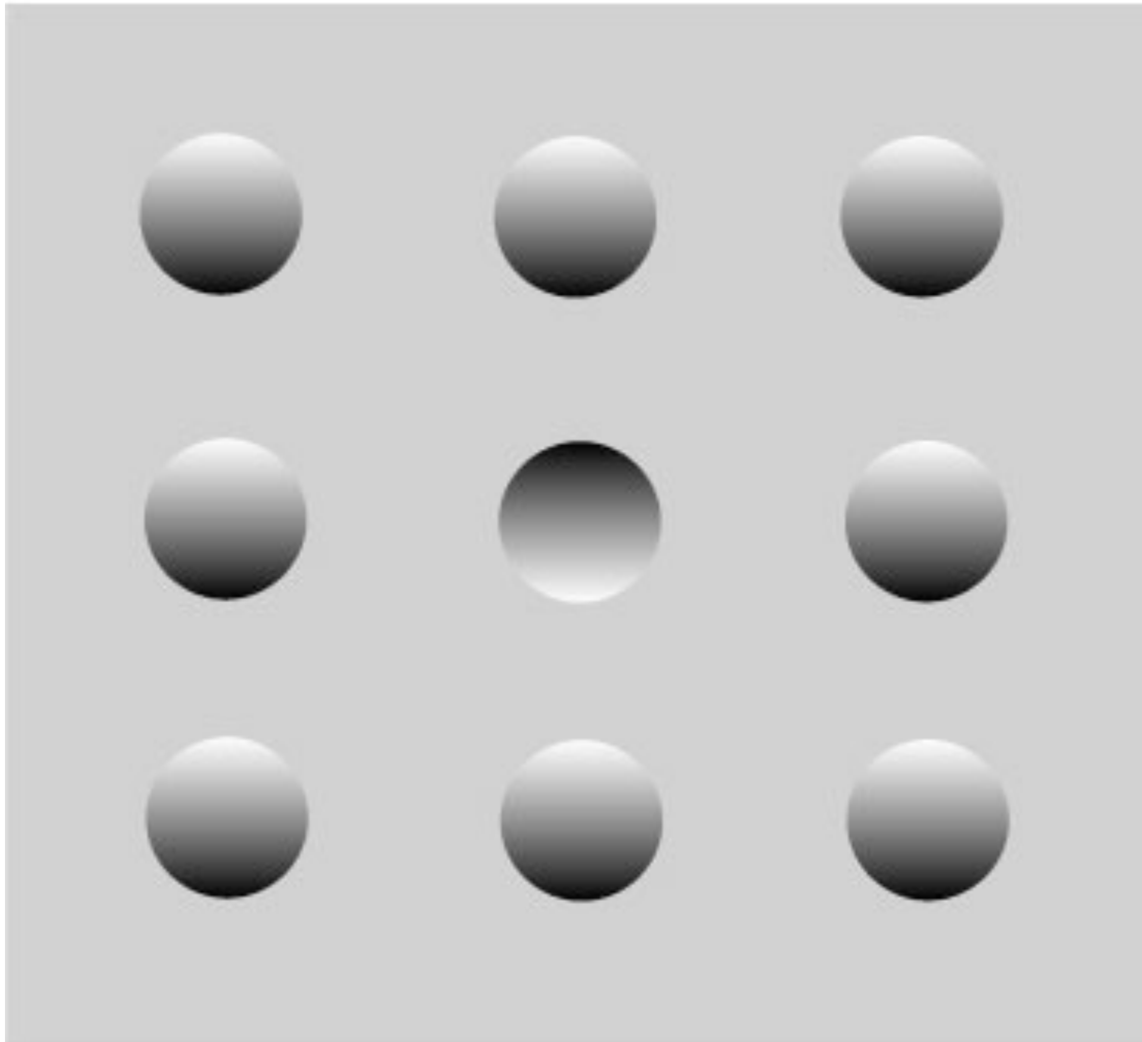
How does our brain go about deciding which interpretation?

P(image | A) and P(image | B) are equal!   (both A and B could have generated this image)

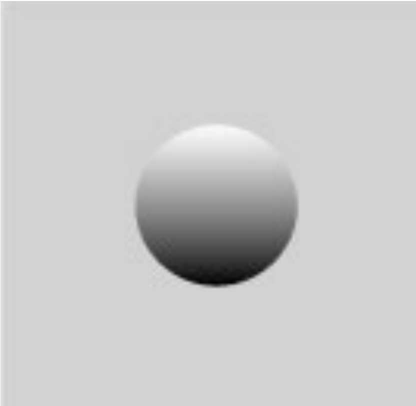
Let's use Bayes' rule:

P(A | image) = P(image | A) P(A)
P(B | image) = P(image | B) P(B)

Which of these is greater?



Is the middle circle popping “out” or “in”?



P(image | OUT & light is above) = 1
P(image | IN & Light is below) = 1

- Image equally likely to be OUT or IN given sensory data alone

What we want to know: P(OUT | image) vs. P(IN | image)

<u>Apply Bayes' rule:</u>

prior

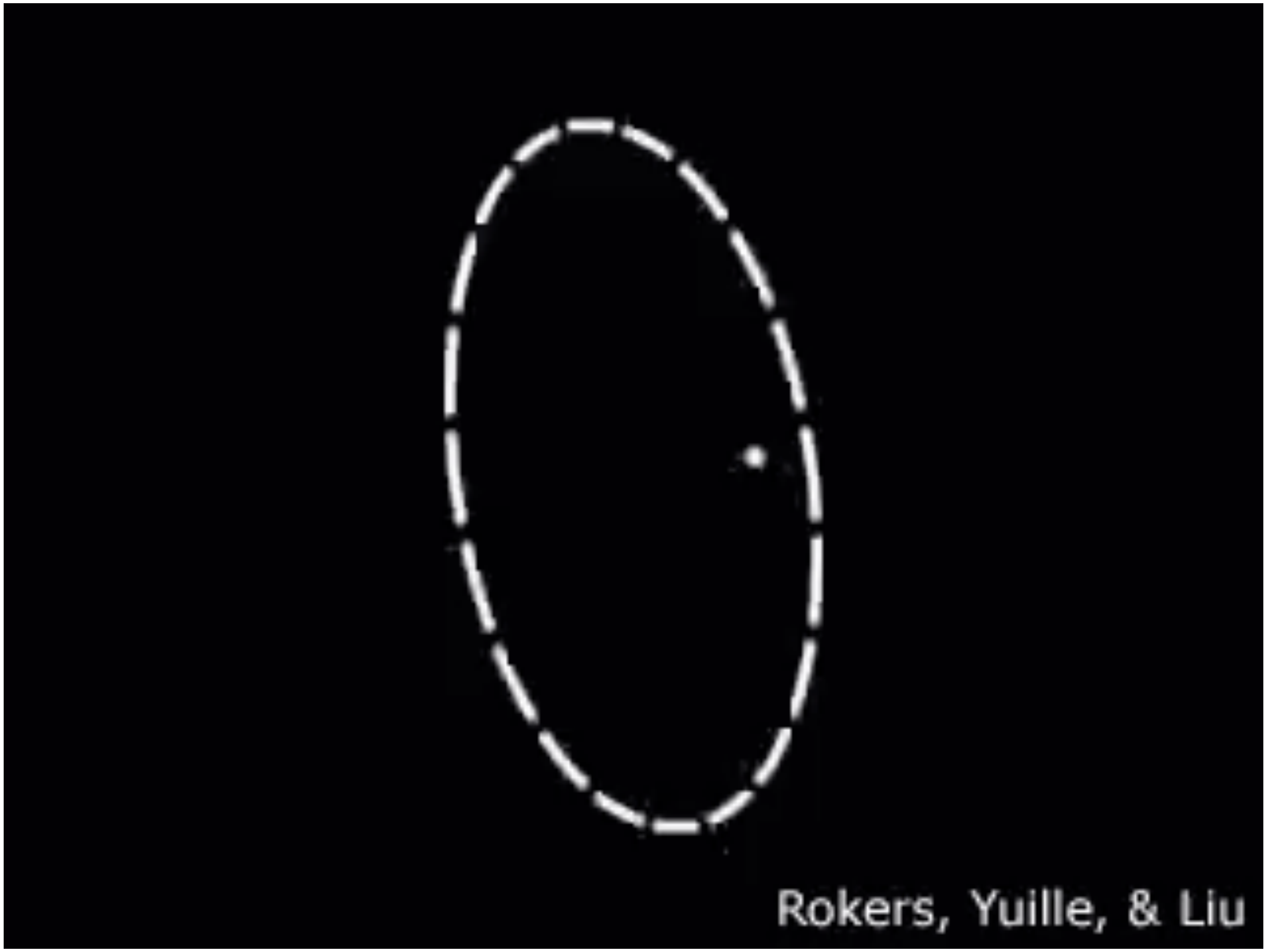
P(OUT | image) ∝ P(image | OUT & light above) × P(OUT) × P(light above)

P(IN | image) ∝ P(image | IN & light below) × P(IN) × P(light below)

Which of these is greater?

Motion example: "stereokinetic effect"

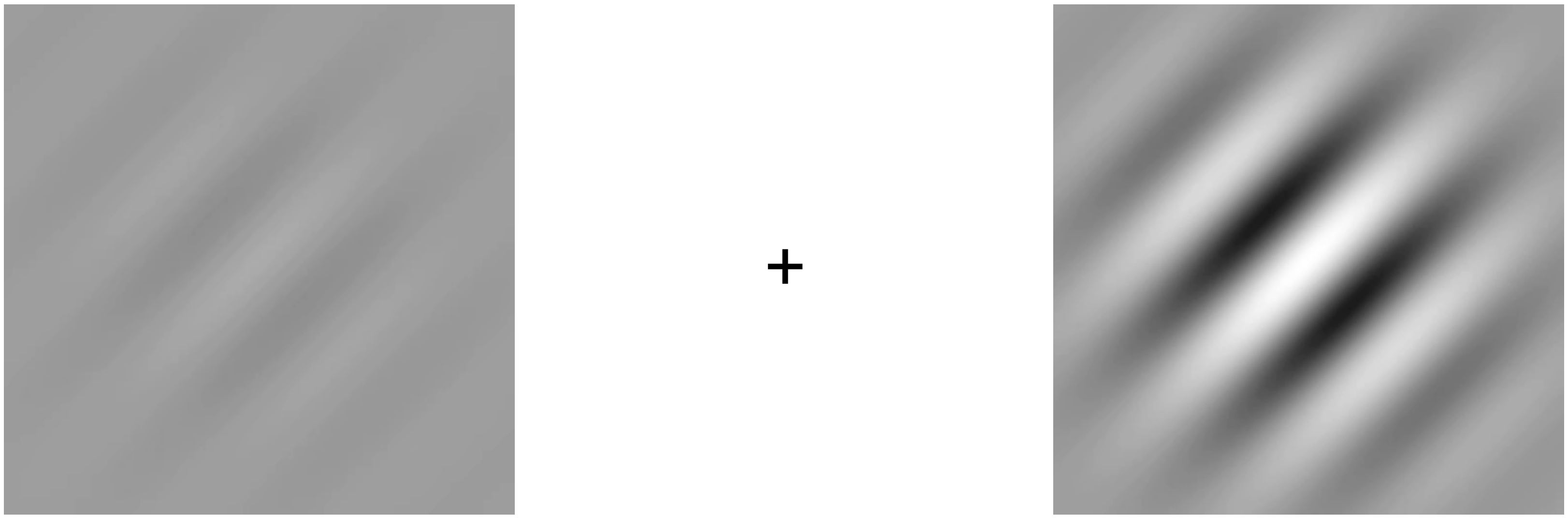
- use prior to interpret ambiguous motions



At least two possible scene interpretations are possible

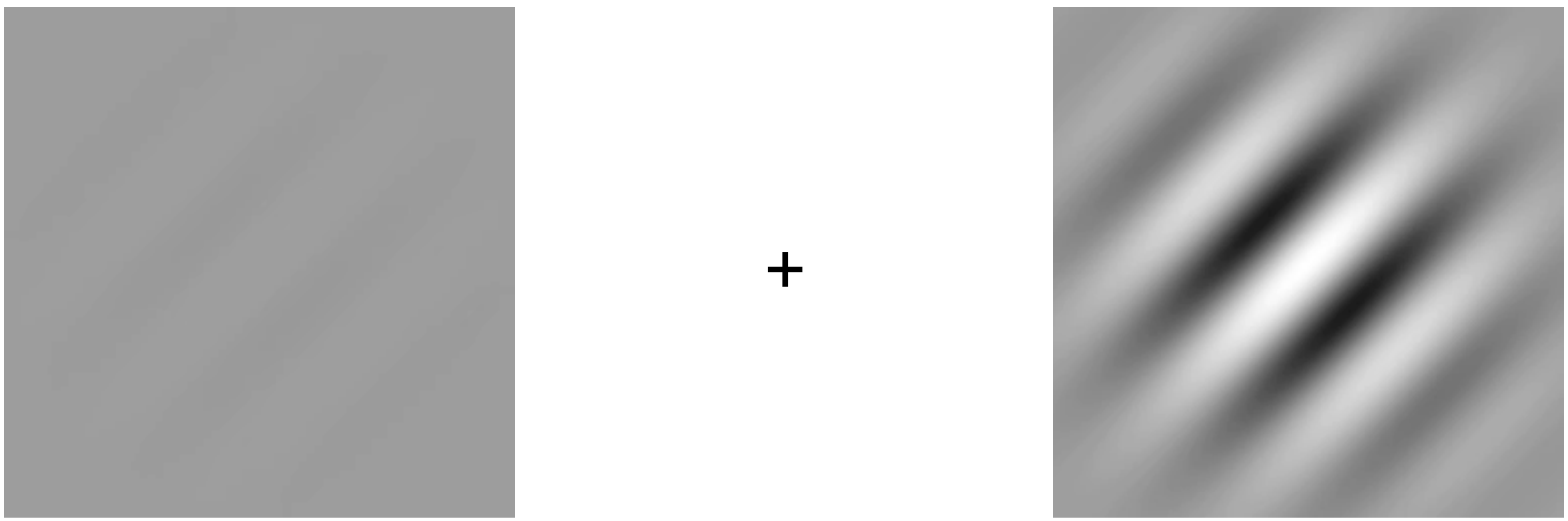
- both could give rise to the same visual input
- percept is therefore determined by which has higher prior of occurring

Application #1: Biases in Motion Perception



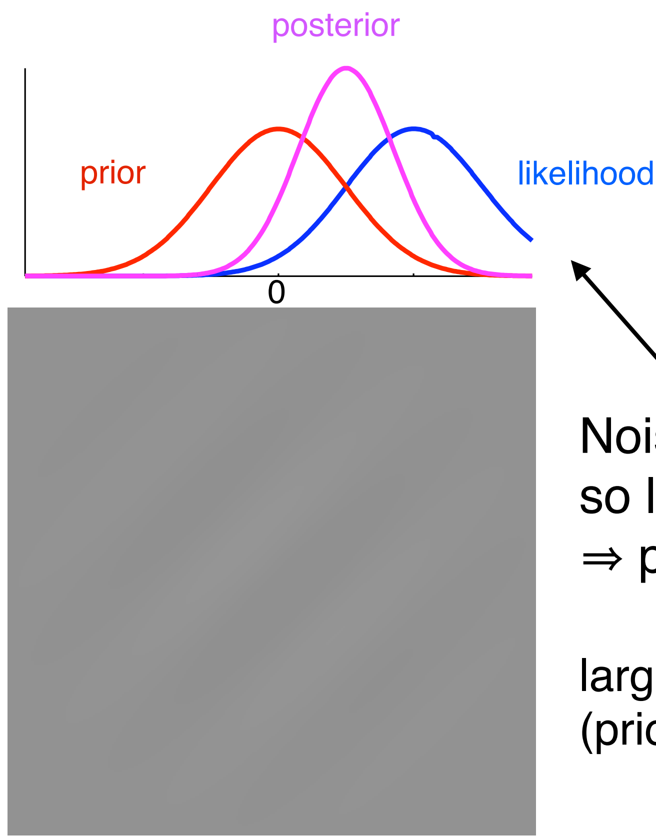
Which grating moves faster?

Application #1: Biases in Motion Perception



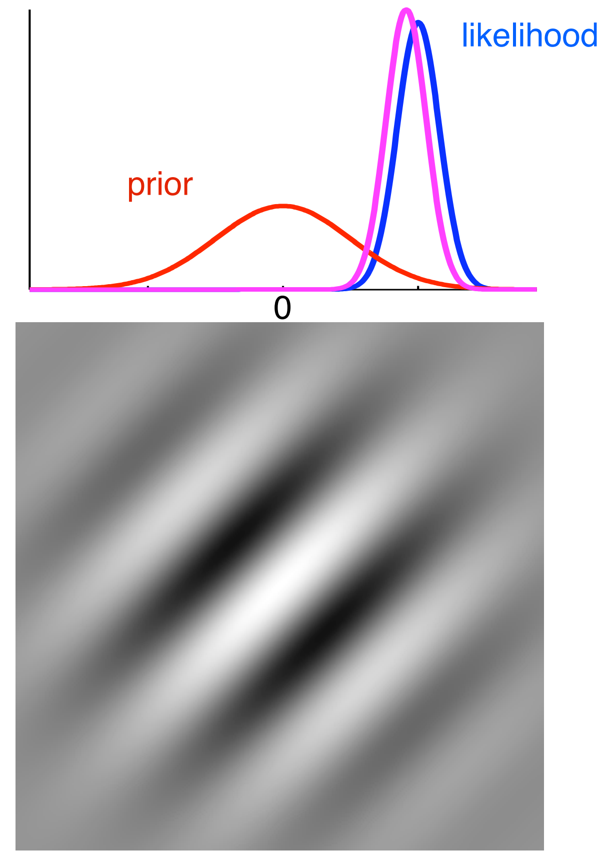
Which grating moves faster?

Explanation from Weiss, Simoncelli & Adelson (2002):



Noisier measurements, so likelihood is broader ⇒ posterior has

larger shift toward 0 (prior = no motion)



- In the limit of a zero-contrast grating, likelihood becomes infinitely broad ⇒ percept goes to zero-motion.

- Claim: explains why people actually speed up when driving in fog!

Hollow Face Illusion

http://www.richardgregory.org/experiments/

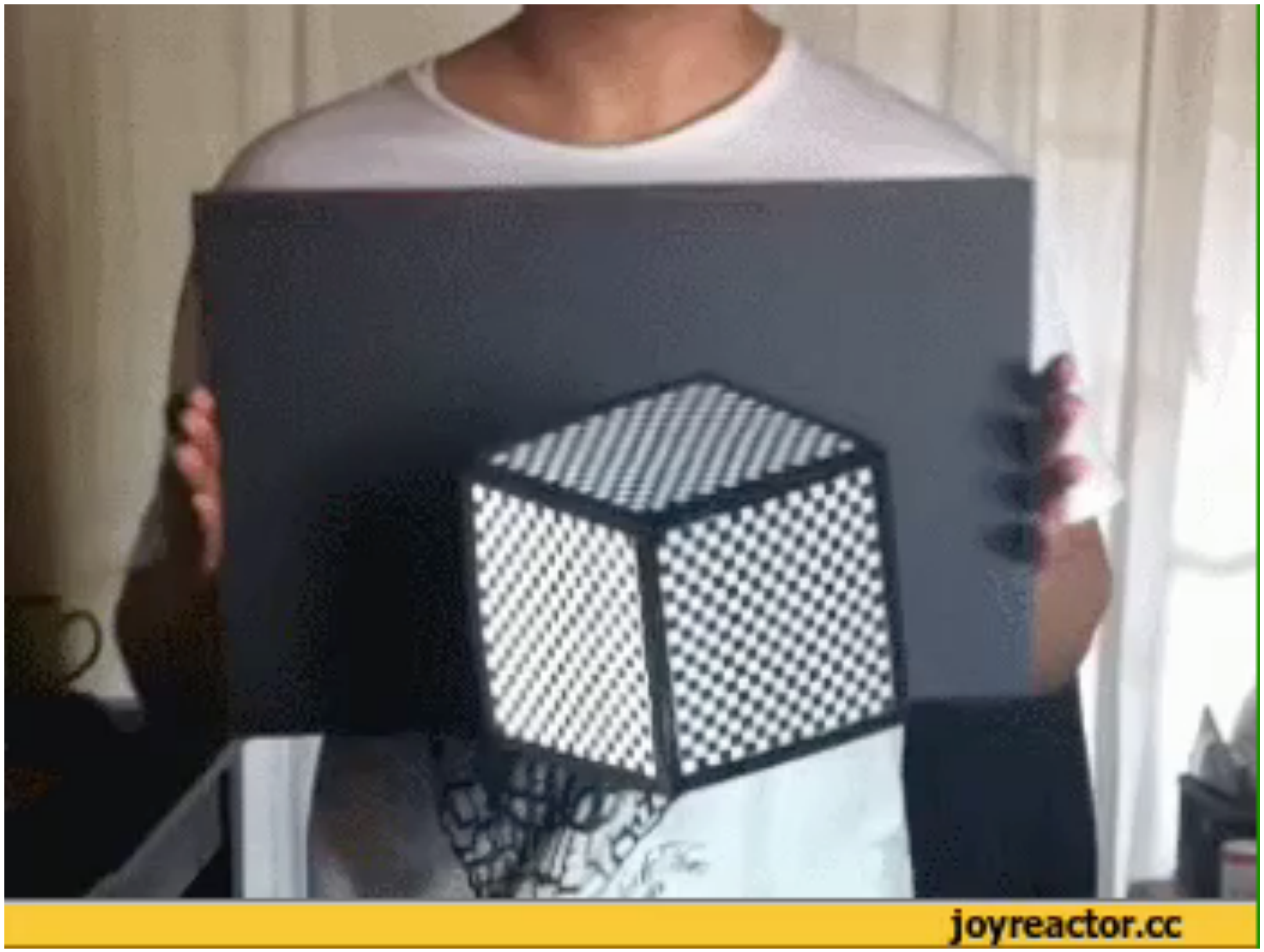
Hollow Face Illusion

Hypothesis #1: face is concave
Hypothesis #2: face is convex

$$P(\text{convex}|\text{video}) \propto P(\text{video}|\text{convex})\, P(\text{convex})$$
$$P(\text{concave}|\text{video}) \propto P(\text{video}|\text{concave})\, P(\text{concave})$$

posterior likelihood prior

$P(\text{convex}) > P(\text{concave}) \Rightarrow$ posterior probability of convex is higher (which determines our percept)


joyreactor.cc

Hollow Face Illusion: other examples

mask with nose ring

http://www.youtube.com/watch?NR=1&v=Rc6LRxjqzkA

this one is so strange it looks incredibly fake, but it's a real video!

Gathering for Gardner dragon

https://www.youtube.com/watch?v=MUZS_UY0pgg

https://www.youtube.com/watch?v=QzggKdkPRc8

You can download this and make one yourself!

- our prior belief that objects are convex is SO strong, we can't over-ride it, even when we know intellectually it's wrong!

Summary:

- Perception is an ill-posed problem
- equivalently: the world is still ambiguous even given all our sensory information
- Probabilistic information can be used to solve ill-posed problems (via Bayes' theorem)
- Bayes' theorem:

$$\overbrace{P(\text{world} \mid \text{sense data})}^{\text{posterior}} \propto \overbrace{P(\text{sense data} \mid \text{world})}^{\text{likelihood}} \; \overbrace{P(\text{world})}^{\text{prior}}$$

- The brain takes into account "prior knowledge" to figure out what's in the world given our sensory information