# Perception: The Bayesian Approach

(Discussed in chapter 6)

Lecture 19

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# Bayesian theories of perception: dealing with probabilities

# Quick math quiz:

$$x + 3 = 8$$

What is x?

# Quick math quiz:

$$x \times 2 = 10$$

What is x?

# Quick math quiz:

$$x + y = 9$$

What are x and y?

This is an example of an *ill-posed* problem

- problem that has no unique solution

# Perception is also an ill-posed problem!

Example #1:

Spectrum of Illuminant $\times$ Reflectance function of surface $=$ Light Hitting Eye

Question we want to answer: what are the surface properties (i.e., color) of the surface?

Equivalently: $X \times Y = R$ (cone responses)

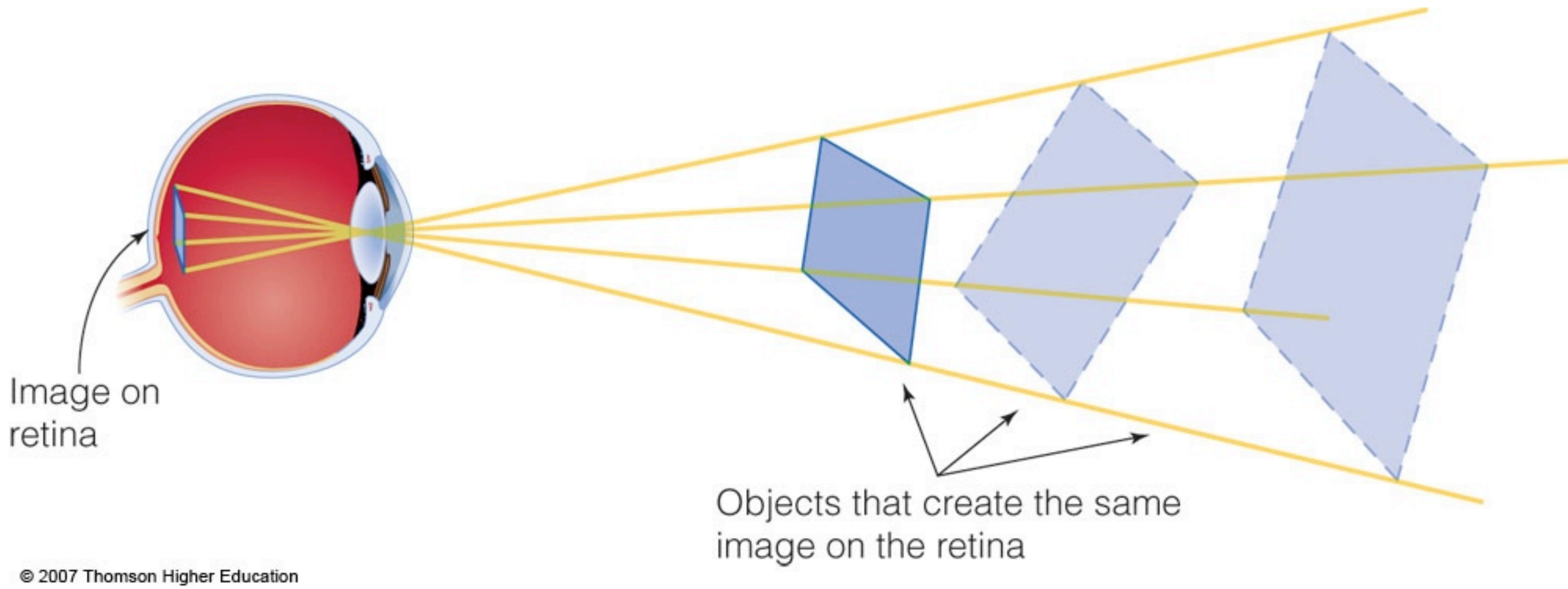
Given R, was Y?

(you'd have to know X to make it well-posed)

# Perception is also an ill-posed problem!

Example #2: 3D world $\Rightarrow$ 2D retinal image

Question: what's out there in the 3D world?




- ill-posed because there are infinitely many 3D objects that give rise to the same 2D retinal image
- need some additional info to make it a *well-posed* problem

Luckily, having some probabilistic information can help:

$$x + y = 9$$

Table showing past values of y:

y

| | | |
|---|---|---|
| 7 | 7 | 7 |
| 7 | 7 | 7 |
| 5 | 7 | 7 |
| 7 | 6 | 7 |
| 7 | 7 | 7 |
| 8 | 7 | 8 |
| 7 | 7 | 7 |
| 7 | 7 | 7 |

Given this information, what would you guess to be the values of x?

How confident are you in your answers?

# A little math: **Bayes' rule**

- very simple formula for manipulating probabilities

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

probability of B

probability of A

conditional probability
"probability of B given that A occurred"

**simplified form:** $$P(B \mid A) \propto P(A \mid B)\, P(B)$$

# A little math: **Bayes' rule**

$$P(B \mid A) \propto P(A \mid B)\, P(B)$$

## Example: 2 coins


- one coin is fake: “heads” on both sides (H / H)
- one coin is standard: (H / T)

You grab one of the coins at random and flip it. It comes up “heads”. What is the probability that you’re holding the fake?

$$p(\text{Fake} \mid H) \propto p(H \mid \text{Fake})\, p(\text{Fake})$$

$$(1) \quad (\tfrac{1}{2}) \quad = \tfrac{1}{2} \qquad \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}} = \frac{2}{3}$$

$$p(\text{Nrml} \mid H) \propto p(H \mid \text{Nrml})\, p(\text{Nrml})$$

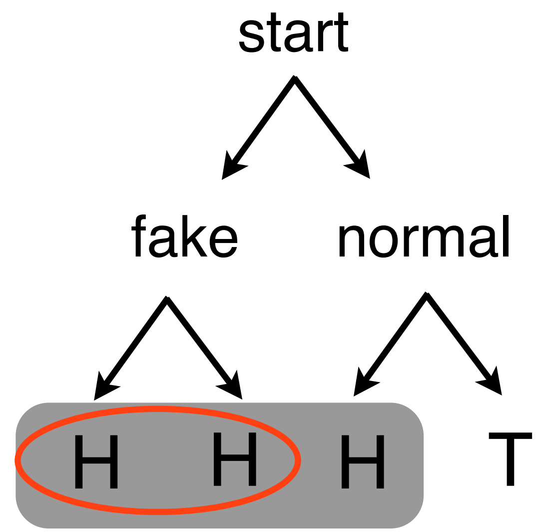
$$(\tfrac{1}{2}) \quad (\tfrac{1}{2}) \quad = \tfrac{1}{4} \qquad \frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{3}$$

probabilities must sum to 1

# A little math: **Bayes' rule**

$$P(B \mid A) \propto P(A \mid B)\, P(B)$$

## Example: 2 coins




$$p(\text{ Fake} \mid H) \propto p(H \mid \text{Fake})\, p(\text{Fake}) \qquad \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}} = \frac{2}{3}$$

$$(\ 1\ ) \quad (\ ½\ ) \quad = ½$$

$$p(\text{ Nrml} \mid H) \propto p\,(H \mid \text{Nrml})\, p(\text{Nrml}) \qquad \frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{3}$$

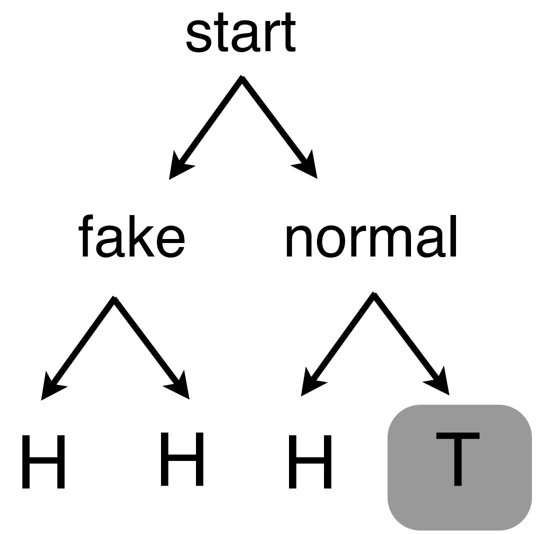
$$(\ ½\ ) \quad (\ ½\ ) \quad = ¼$$

probabilities must sum to 1

# A little math: **Bayes' rule**

$$P(B \mid A) \propto P(A \mid B)\, P(B)$$

## Example: 2 coins




Experiment #2: It comes up "tails".
What is the probability that you're holding the fake?

$$p(\text{Fake} \mid T) \propto p(T \mid \text{Fake})\, p(\text{Fake})$$

$$(0) \quad (½) \quad = 0$$

$$p(\text{Nrml} \mid T) \propto p(T \mid \text{Nrml})\, p(\text{Nrml})$$

$$(½) \quad (½) \quad = ¼$$

probabilities must sum to 1

$= 0$

$= 1$

# What does this have to do with perception?

**Bayes' rule:** $P(B \mid A) \propto P(A \mid B)\, P(B)$

Formula for computing: P(what's in the world | sensory data)

(This is what our brain wants to know!)

B A

P(world | sense data) ∝ P(sense data | world) P(world)

Posterior

(resulting beliefs about the world)

Likelihood

(given by laws of physics; ambiguous because many world states could give rise to same sense data)

Prior

(given by past experience)

# Helmholtz: perception as "optimal inference"

helmholtz 1821-1894

"Perception is our best guess as to what is in the world, given our current sensory evidence and our prior experience."

$$P(\text{world} \mid \text{sense data}) \propto P(\text{sense data} \mid \text{world})\, P(\text{world})$$

**Posterior**

(resulting beliefs about the world)

**Likelihood**

(given by laws of physics; ambiguous because many world states could give rise to same sense data)

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(given by past experience)

# Helmholtz: perception as "optimal inference"

helmholtz 1821-1894

"Perception is our **best guess** as to what is in the world, given our current **sensory evidence** and our **prior experience**."

$$P(\text{world} \mid \text{sense data}) \propto P(\text{sense data} \mid \text{world})\, P(\text{world})$$

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(resulting beliefs about the world)

Likelihood

(given by laws of physics; ambiguous because many world states could give rise to same sense data)

Prior

(given by past experience)

# what is perception?

**prior** ("top down")

statistical knowledge
about the structure
of the world

- seeing
- hearing
- touching
- smelling
- tasting
- orienting

*percept*

**posterior**

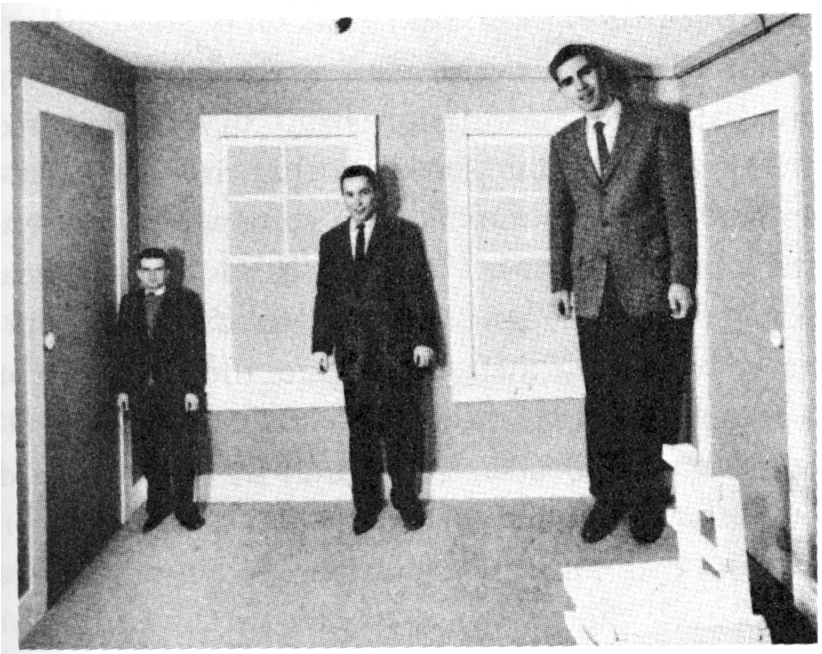
**likelihood** ("bottom up")

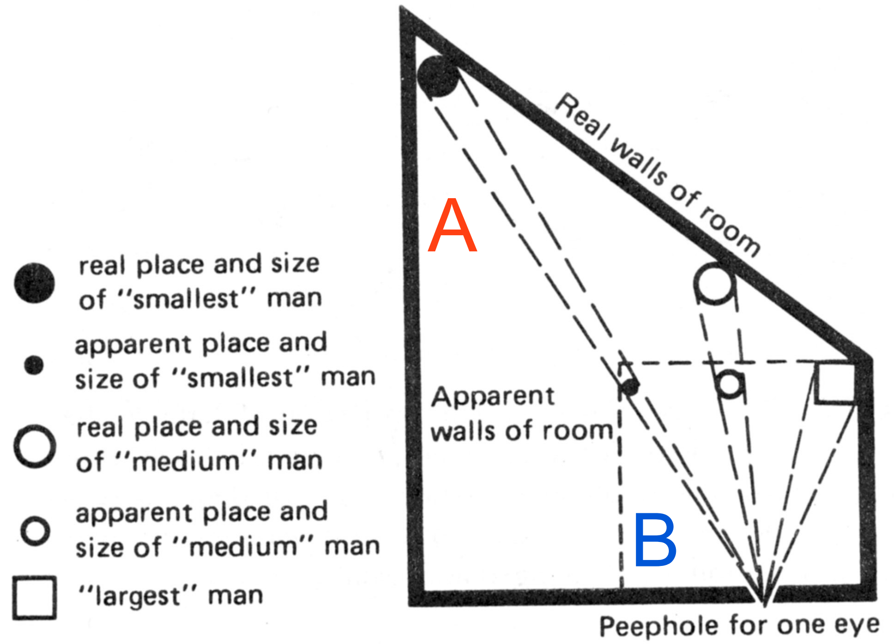
# Examples

Using Bayes' rule to understand how the brain resolves ambiguous stimuli

# Many different 3D scenes can give rise to the same 2D retinal image

The Ames Room





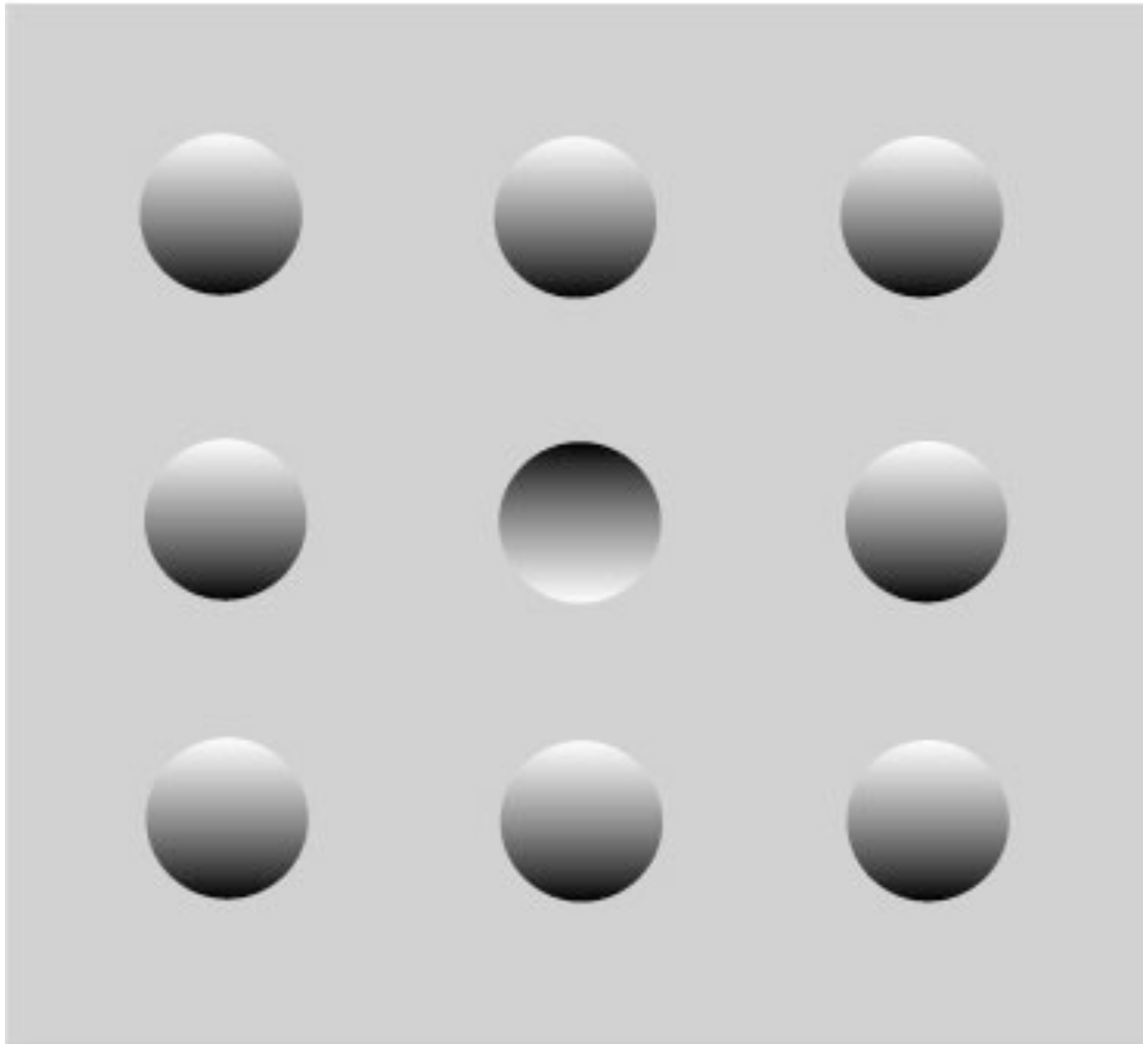

## How does our brain go about deciding which interpretation?

P(image | A) and P(image | B) are equal! (both A and B could have generated this image)

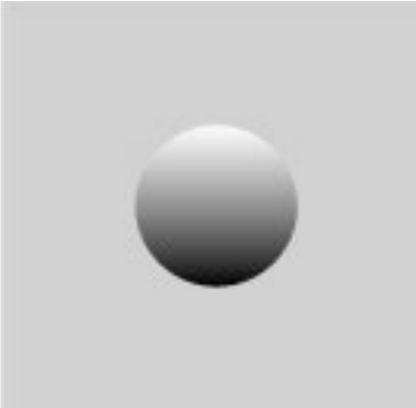
Let's use Bayes' rule:

P(A | image) = P(image | A) P(A)
P(B | image) = P(image | B) P(B)

Which of these is greater?



Is the middle circle popping “out” or “in”?



$P(\text{ image} \mid \text{OUT} \ \&\ \text{light is above}) = 1$

$P(\text{image} \mid \text{IN} \ \&\ \text{Light is below}) = 1$

- Image equally likely to be OUT or IN given sensory data alone

What we want to know: $P(\text{OUT} \mid \text{image})$ vs. $P(\text{IN} \mid \text{image})$

<u>Apply Bayes' rule:</u>

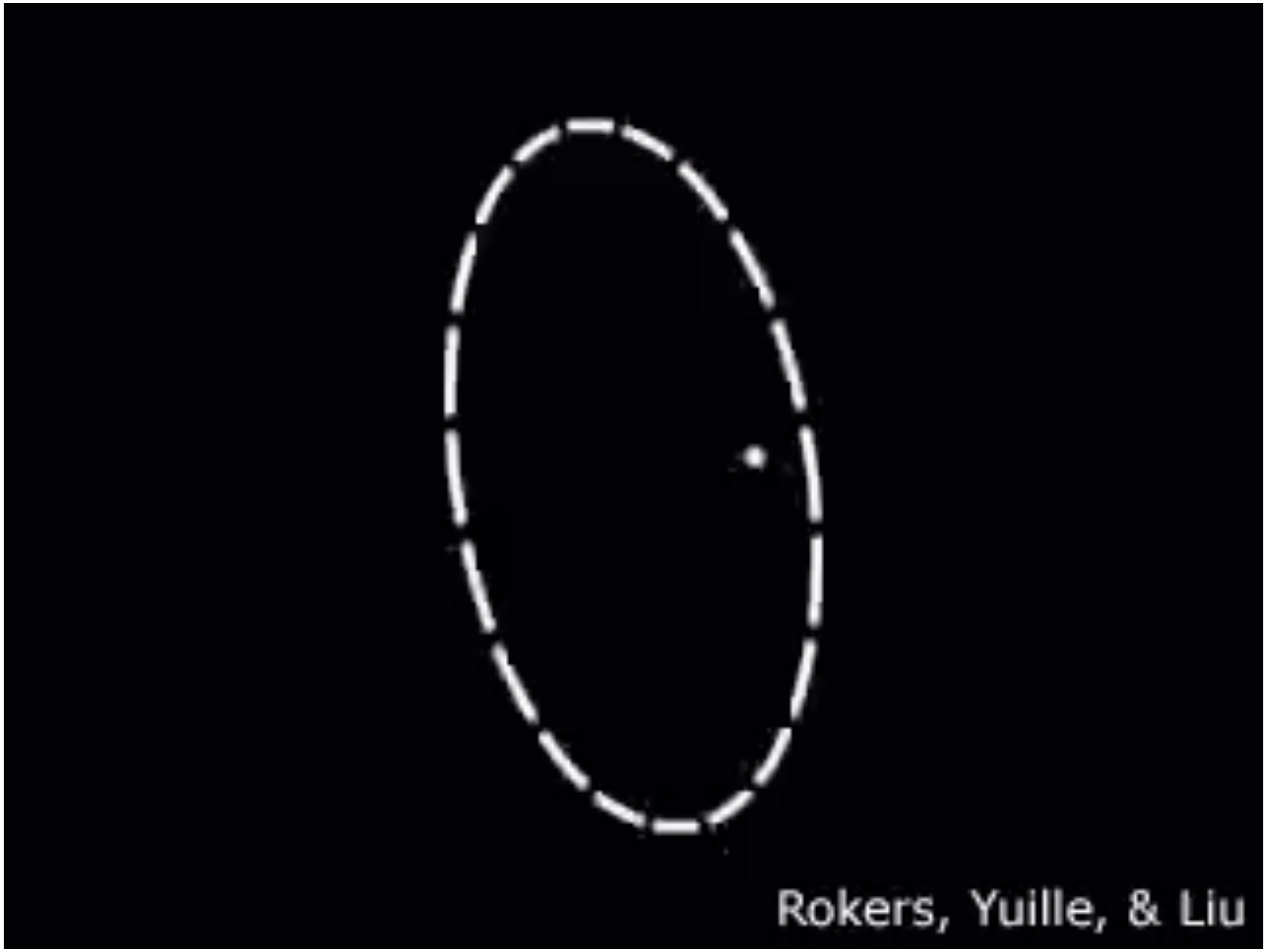
$$P(\text{OUT} \mid \text{image}) \propto P(\text{image} \mid \text{OUT} \ \&\ \text{light above}) \times \overbrace{P(\text{OUT}) \times P(\text{light above})}^{\text{prior}}$$

$$P(\text{IN} \mid \text{image}) \propto P(\text{image} \mid \text{IN} \ \&\ \text{light below}) \times P(\text{IN}) \times P(\text{light below})$$

Which of these is greater?

**Motion example**: "stereokinetic effect"

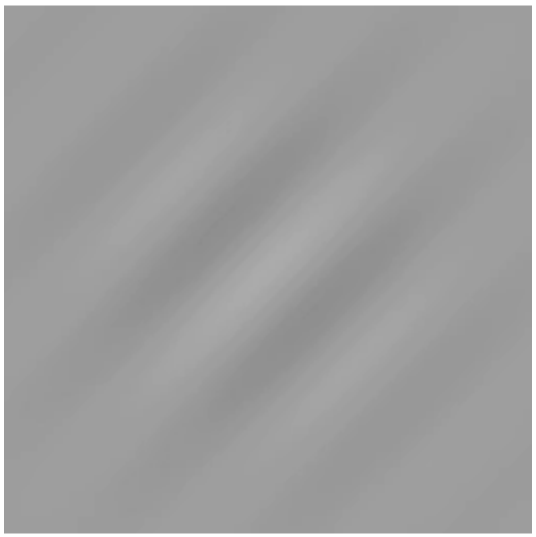
- use prior to interpret ambiguous motions




At least two possible scene interpretations are possible

- both could give rise to the same visual input
- percept is therefore determined by which has higher prior of occurring

# Application #1: Biases in Motion Perception



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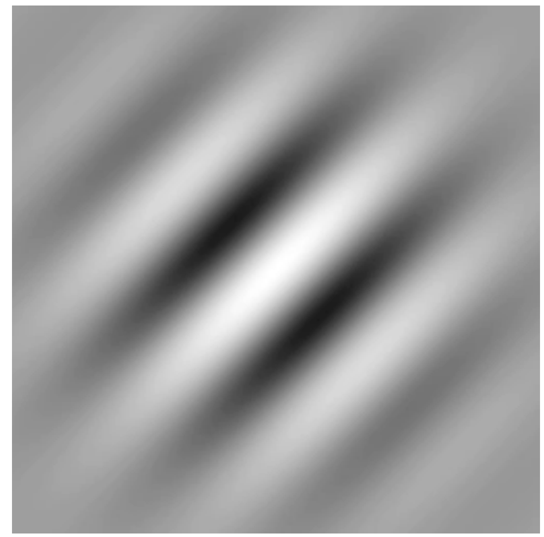

Which grating moves faster?

# Application #1: Biases in Motion Perception

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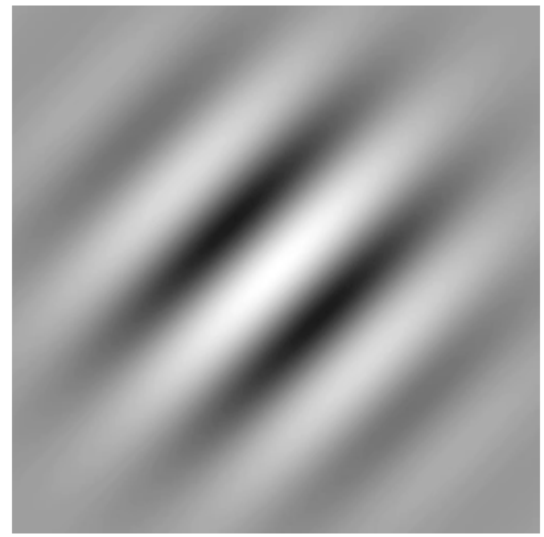
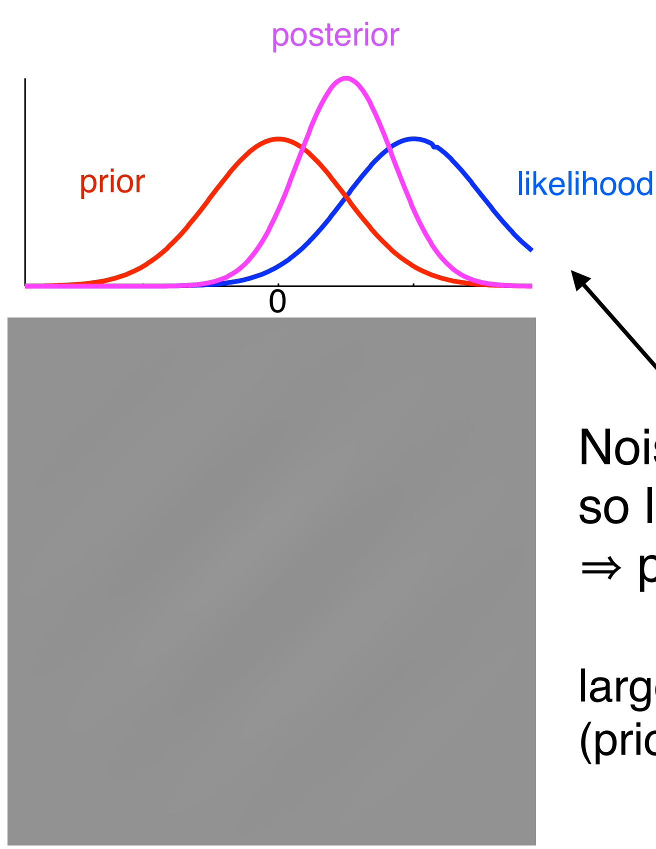


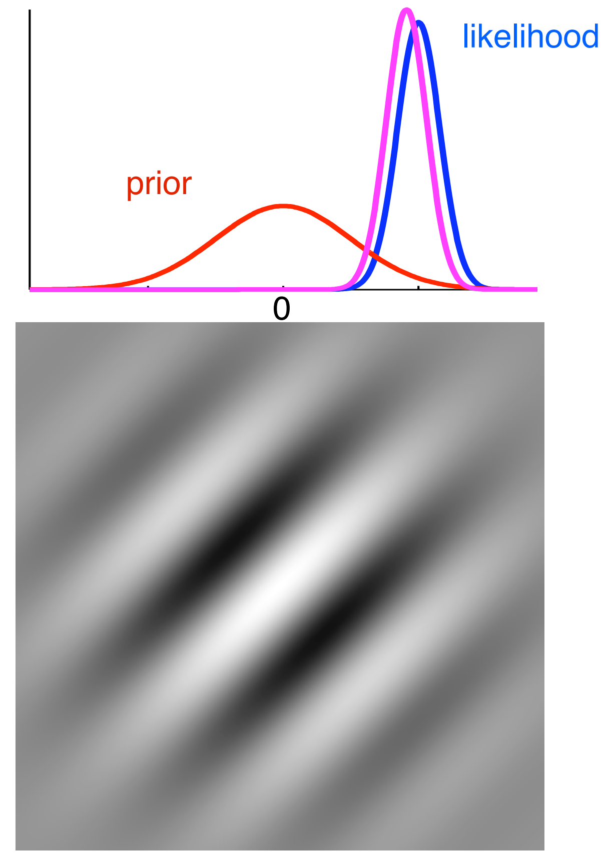
Which grating moves faster?

# Explanation from Weiss, Simoncelli & Adelson (2002):




Noisier measurements, so likelihood is broader ⇒ posterior has

larger shift toward 0
(prior = no motion)




- In the limit of a zero-contrast grating, likelihood becomes infinitely broad ⇒ percept goes to zero-motion.

- Claim: explains why people actually speed up when driving in fog!

# Hollow Face Illusion

http://www.richardgregory.org/experiments/

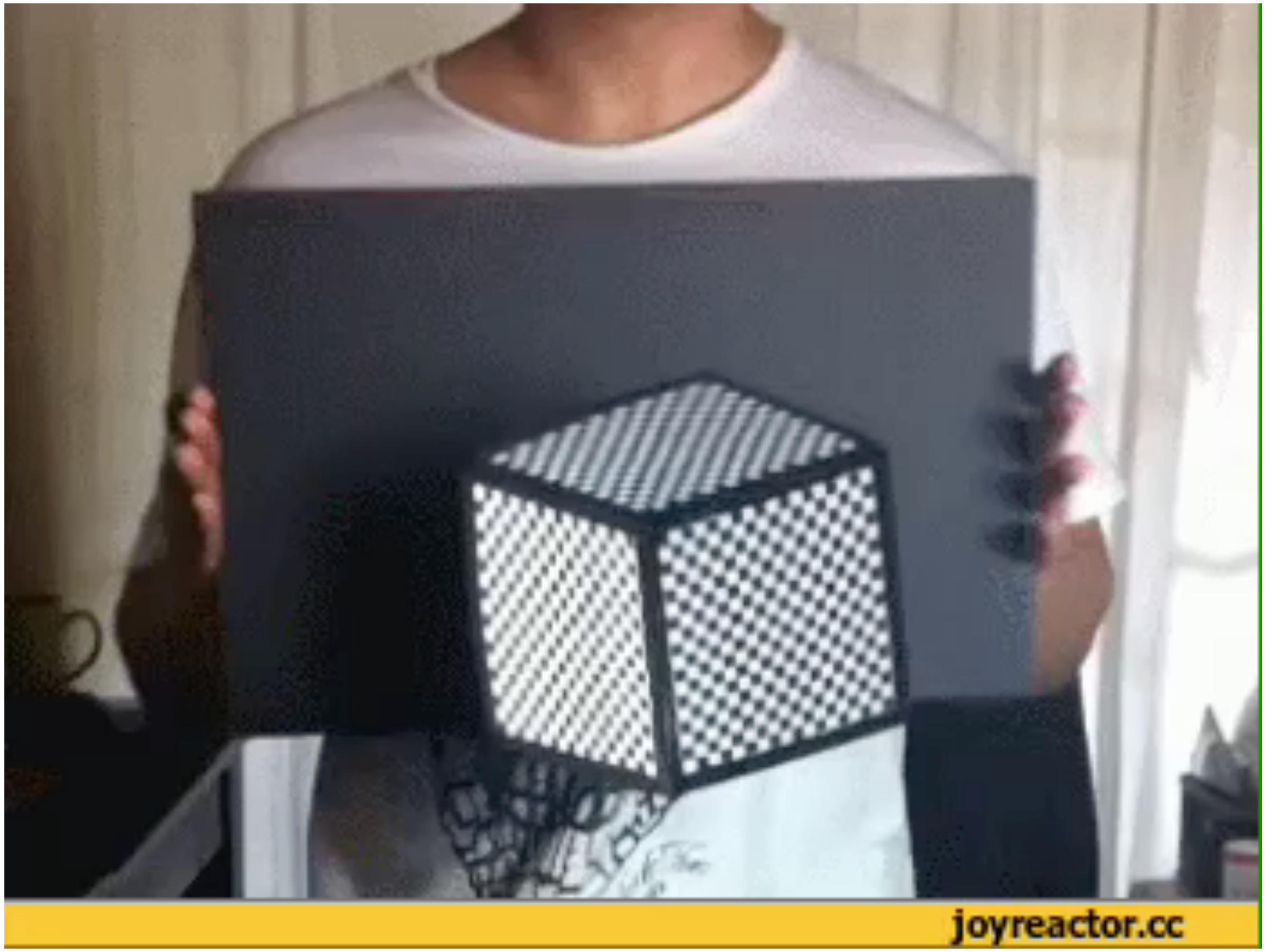
# Hollow Face Illusion

Hypothesis #1: face is concave
Hypothesis #2: face is convex

$$P(\text{convex}|\text{video}) \propto P(\text{video}|\text{convex})\ P(\text{convex})$$
$$P(\text{concave}|\text{video}) \propto P(\text{video}|\text{concave})\ P(\text{concave})$$

posterior likelihood prior

$P(\text{convex}) > P(\text{concave}) \Rightarrow$ posterior probability of convex is higher (which determines our percept)


joyreactor.cc

# Hollow Face Illusion: other examples

mask with nose ring

http://www.youtube.com/watch?NR=1&v=Rc6LRxjqzkA

this one is so strange it looks incredibly fake, but it’s a real video!

Gathering for Gardner dragon


https://www.youtube.com/watch?v=MUZS_UY0pgg

https://www.youtube.com/watch?v=QzggKdkPRc8

You can download this and make one yourself!

- our prior belief that objects are convex is SO strong, we can’t over-ride it, even when we know intellectually it’s wrong!

# Summary:

- Perception is an ill-posed problem
- equivalently: the world is still ambiguous even given all our sensory information
- Probabilistic information can be used to solve ill-posed problems (via Bayes' theorem)
- Bayes' theorem:

$$\underbrace{P(\text{world} \mid \text{sense data})}_{\text{posterior}} \propto \underbrace{P(\text{sense data} \mid \text{world})}_{\text{likelihood}} \underbrace{P(\text{world})}_{\text{prior}}$$

- The brain takes into account "prior knowledge" to figure out what's in the world given our sensory information