Multi-dimensional dynamics

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lecture 23

Review: Dynamical System in one variable

$x(t)$ 1-dimensional quantity that evolves in time

How the variable evolves is described by a **dynamics equation** (also known as an **ordinary differential equation**):

$$\frac{dx}{dt} = f(x)$$

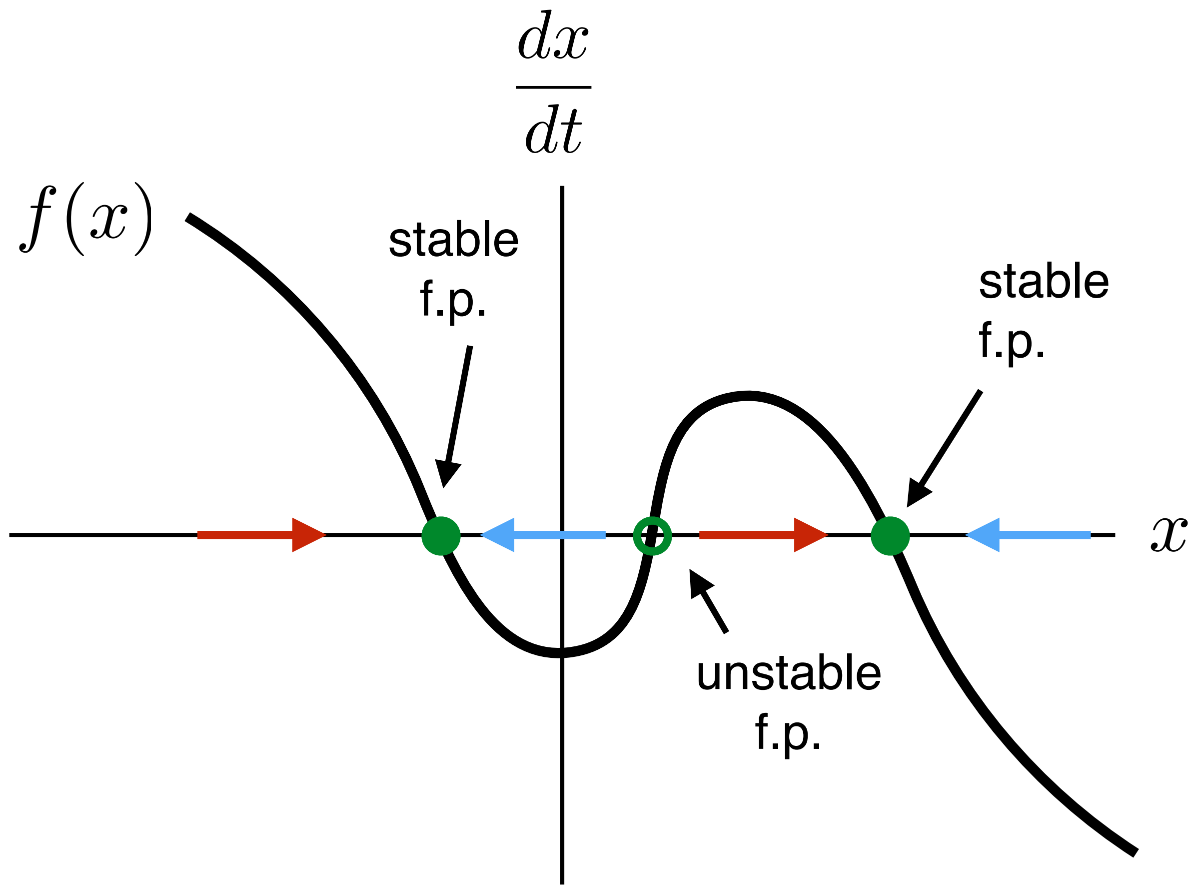
rate of change of x
(ie time-derivative of x)

some function of x

equivalent way of writing it: $\dot{x} = f(x)$

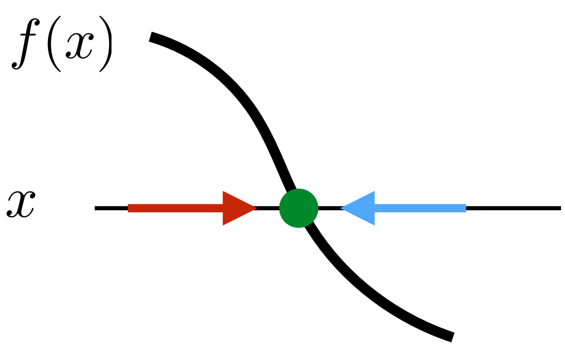
fixed point: location where f(x) = 0

- **stable fixed point**: nearby points converge
- **unstable fixed point**: nearby points diverge

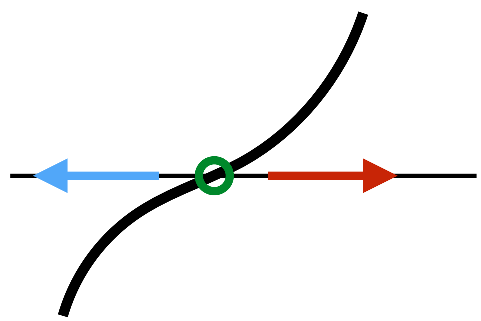


Fixed Point Stability

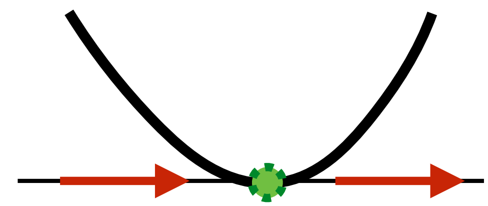
• f(x) crosses x axis with negative slope $\Longrightarrow$ **stable**

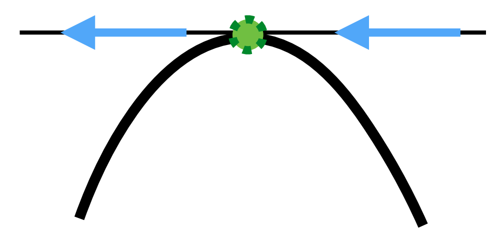


• f(x) crosses x axis with positive slope $\Longrightarrow$ **unstable**



• f(x) touches x axis without crossing: *neither*





other topics

- “solving” a differential equation
- Euler integration
- linear dynamical systems in 1 variable

Dynamical models in neuroscience

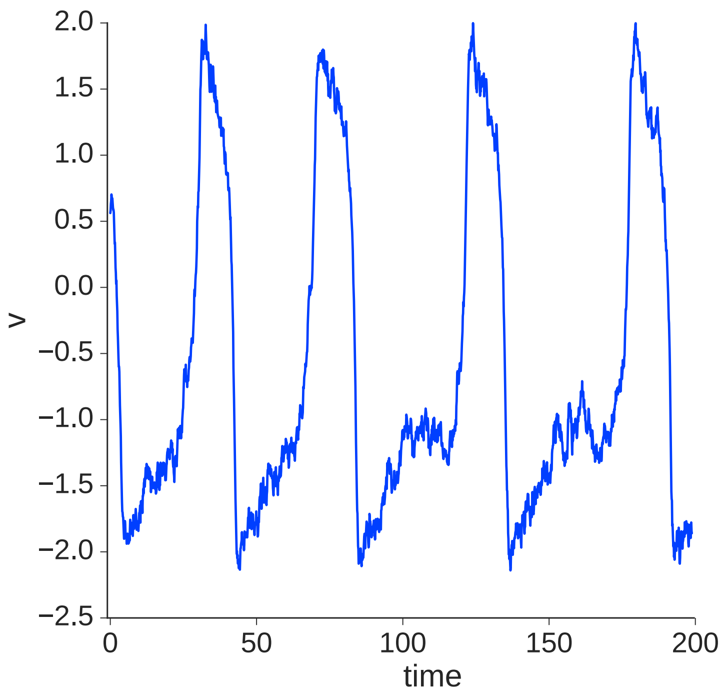
FitzHugh-Nagumo oscillator

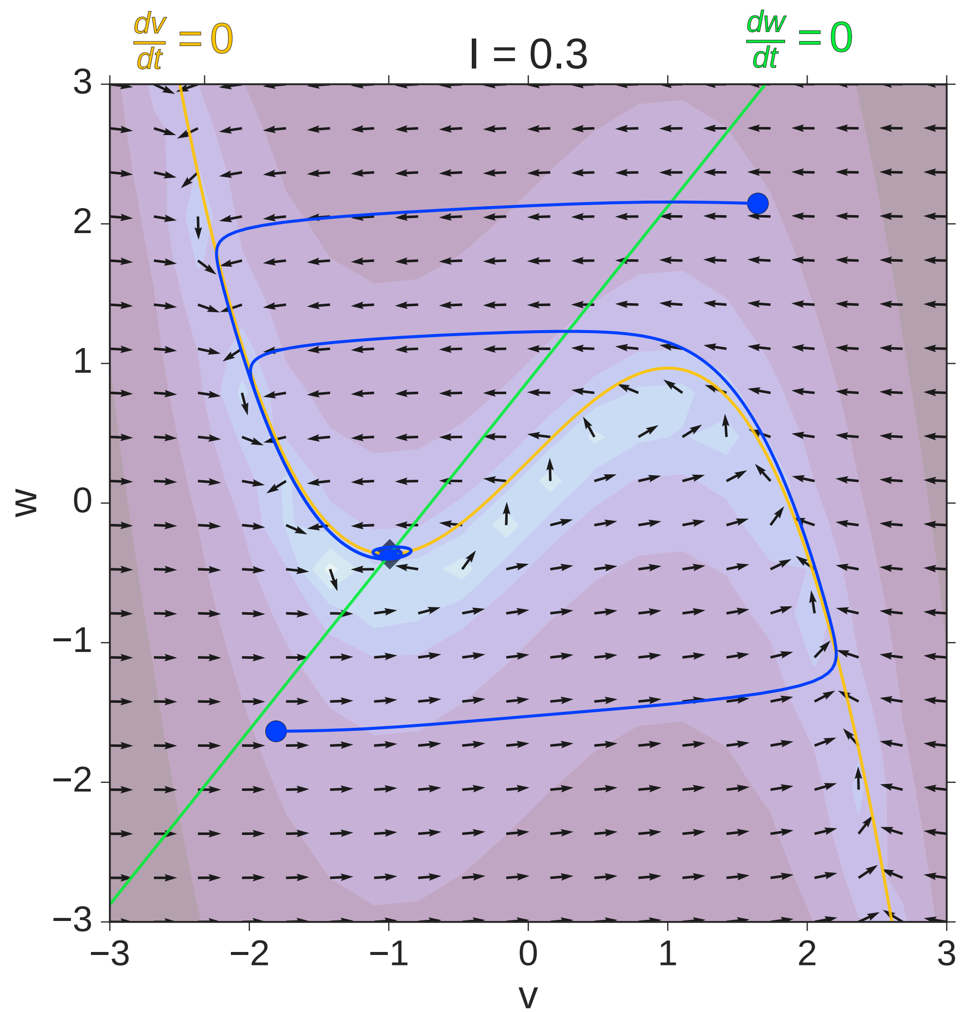
$$\dot{v} = v - \frac{1}{3}v^3 - w + I$$

$$\dot{w} = 0.08(v + 0.7 - 0.8w)$$

limit cycle

Used as a model of cortical up-down states [Curto et al. 2009]



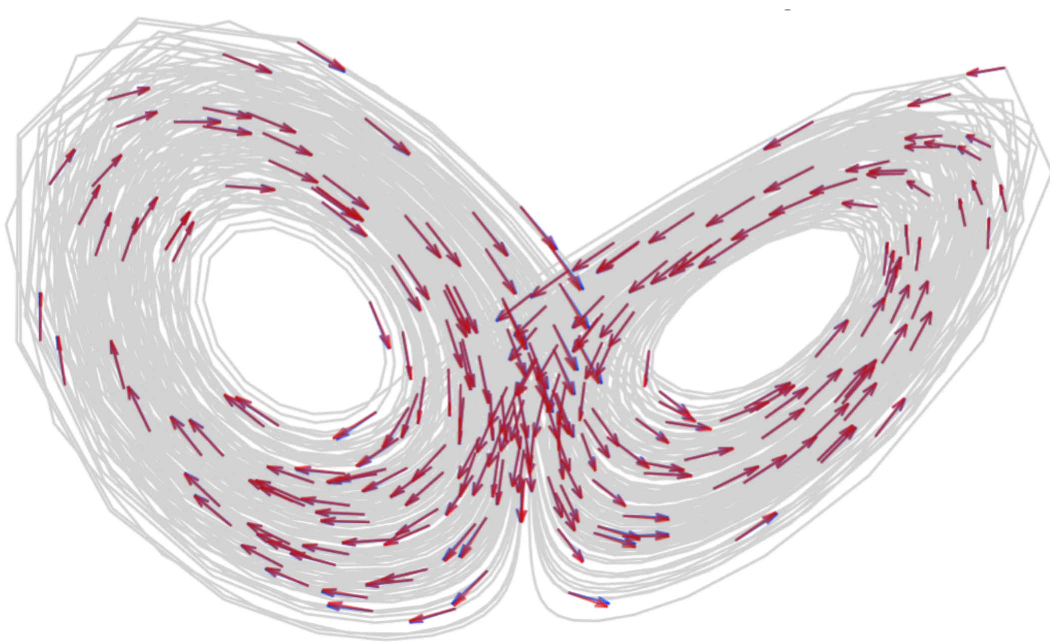


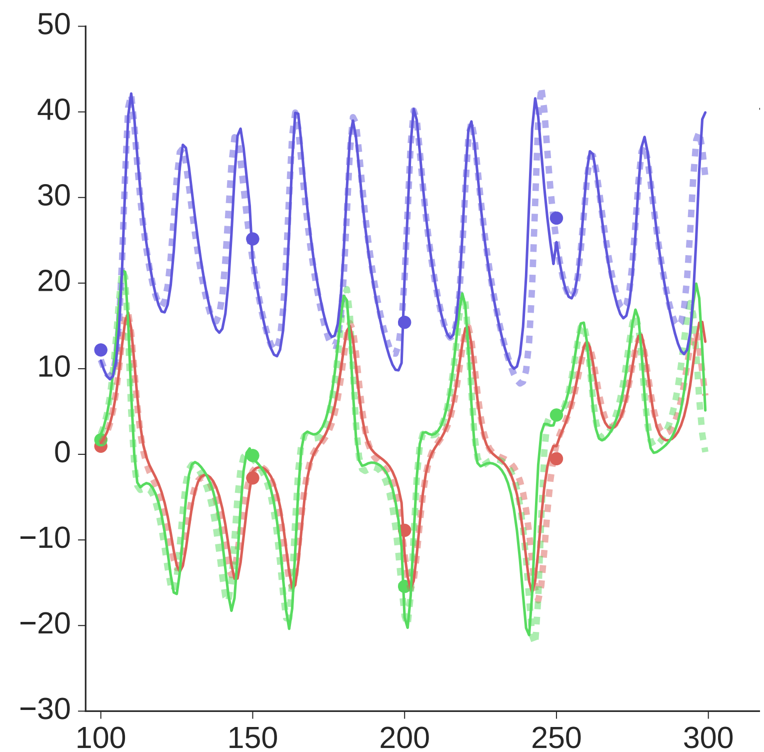
Lorenz attractor

$$\dot{x} = 10(y - x)$$

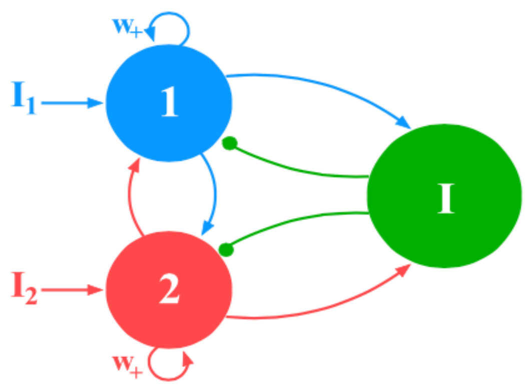
$$\dot{y} = x(28 - z) - y$$

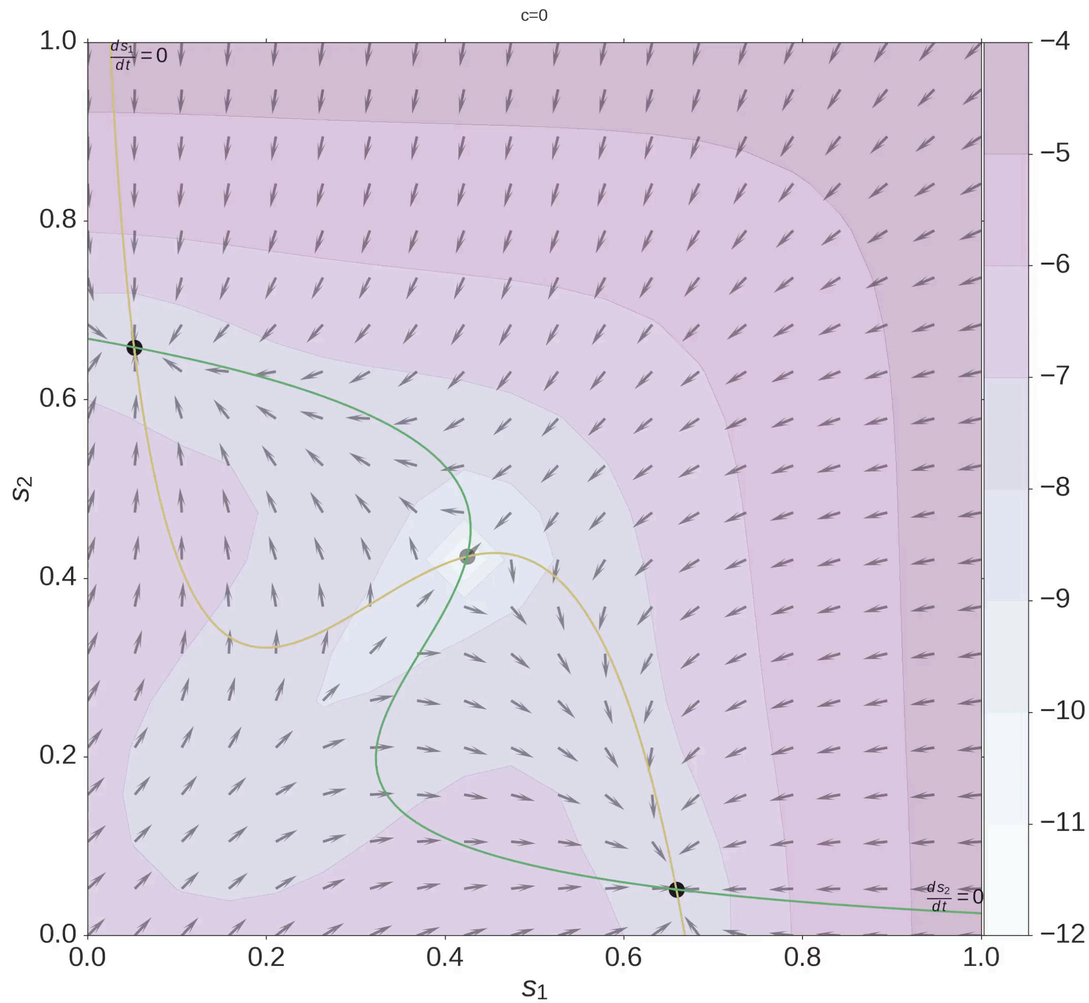
$$\dot{z} = xy - \frac{8}{3}z$$





Perceptual decision-making





Wong, K.-F. and Wang, X.-J. (2006). A recurrent network mechanism of time integration in perceptual decisions. The Journal of Neuroscience, 26(4):1314-1328.

Rotational dynamics in M1

Churchland et al, Nature 2003

Rotational dynamics in V1

https://www.youtube.com/watch?v=CrY5AfNH1ik