# Linear Algebra II:
# linear combinations & matrices

Math Tools for Neuroscience (NEU 314)
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Lecture 3
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accompanying notes/slides

# Linear algebra

"Linear algebra has become as basic and as applicable as calculus, and fortunately it is easier."

- Glibert Strang, *Linear algebra and its applications*

# today's topics

- linear projection (review)
- orthogonality (review)

- linear combination
- linear independence / dependence
- matrix operations: transpose, multiplication, inverse

Did not get to:

- vector space
- subspace
- basis
- orthonormal basis

# Linear Projection Exercise

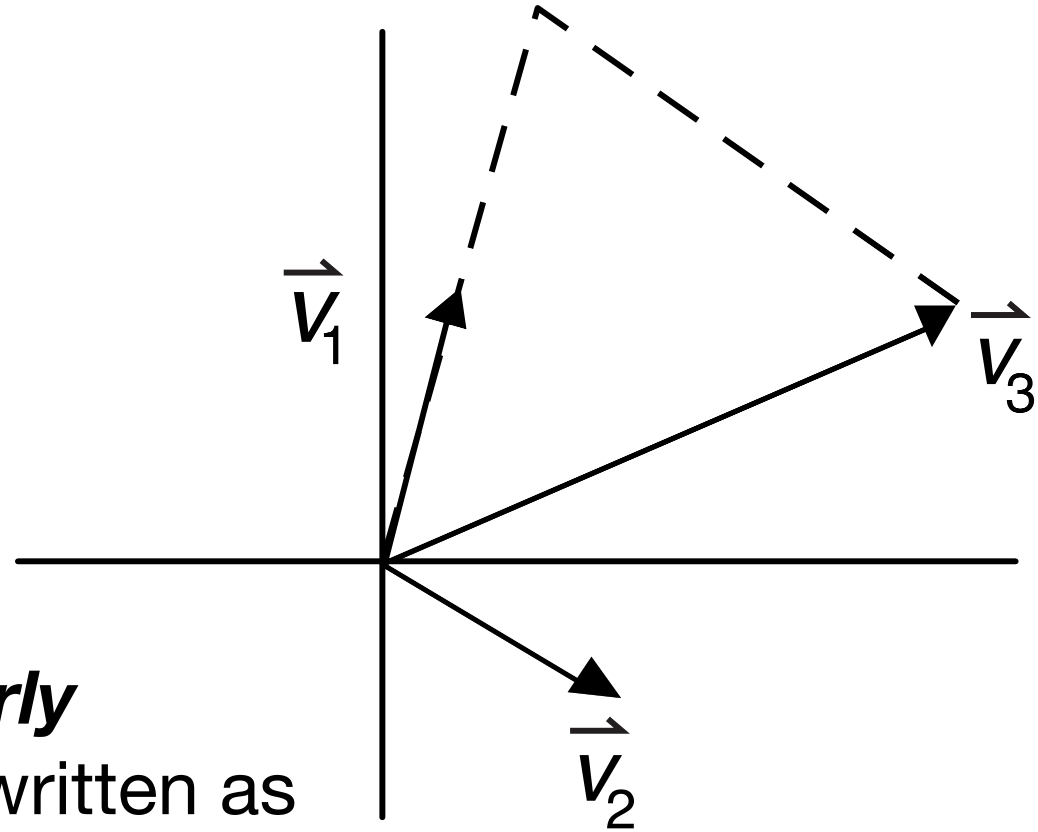
w = [2,2]
v1 = [2,1]
v2 = [5,0]

Compute:
Linear projection of w onto lines defined by v1 and v2

# linear combination

- scaling and summing applied to a group of vectors

$$a\vec{v}_1 + b\vec{v}_2 = \vec{v}_3$$




- a group of vectors is ***linearly dependent*** if one can be written as a linear combination of the others
- otherwise, ***linearly independent***

# matrices

$n \times m$ matrix

$$W = \begin{pmatrix} w_{11} & \cdots & w_{1m} \\ \vdots & & \vdots \\ w_{n1} & \cdots & w_{nm} \end{pmatrix}$$

can think of it as:

m column vectors

$$\begin{pmatrix} | & & | \\ c_1 & \cdots & c_m \\ | & & | \end{pmatrix}$$

or

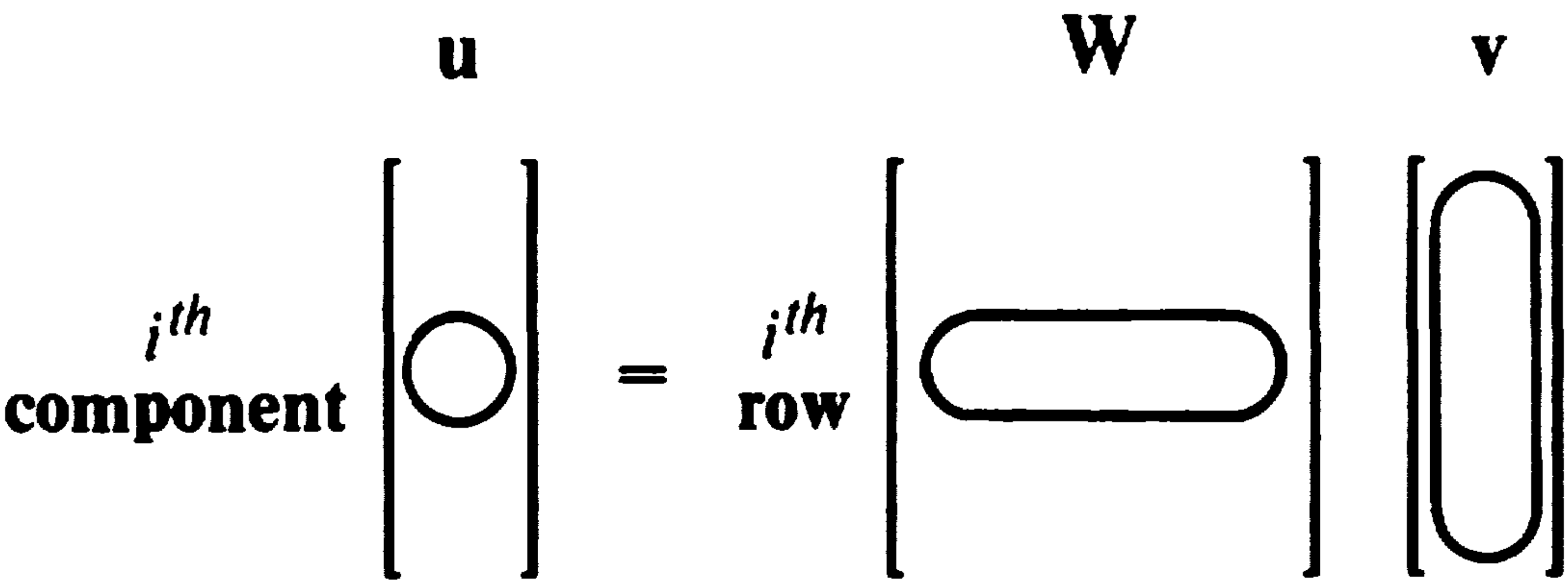
n row vectors

$$\begin{pmatrix} \text{———} \; r_1 \; \text{———} \\ \vdots \\ \text{———} \; r_n \; \text{———} \end{pmatrix}$$

# matrix multiplication

$$\vec{u} = W\vec{v}$$

One perspective: dot product with each row:



# matrix multiplication

Other perspective: *linear combination of columns*

$$\underbrace{\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}}_{\vec{\mathbf{u}}} = \underbrace{\begin{bmatrix} | & & | \\ \vec{c}_1 & \cdots & \vec{c}_m \\ | & & | \end{bmatrix}}_{\mathbf{W}} \underbrace{\begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}}_{\vec{\mathbf{v}}}$$

$$= v_1 \cdot \begin{bmatrix} | \\ \vec{c}_1 \\ | \end{bmatrix} + v_2 \cdot \begin{bmatrix} | \\ \vec{c}_2 \\ | \end{bmatrix} + \ldots + v_m \cdot \begin{bmatrix} | \\ \vec{c}_m \\ | \end{bmatrix}$$

# transpose

- flipping around the diagonal

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

square matrix

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

non-square

- transpose of a product $(AB)^T = B^T A^T$

# inverse

- If A is a square matrix, its inverse $A^{-1}$ (if it exists) obeys

$$AA^{-1} = A^{-1}A = I$$

- inverse of a product $(AB)^{-1} = B^{-1}A^{-1}$