Math Tools for Neuroscience (NEU 314)
Fall 2016

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Lecture 2
(Tuesday 9/20)

accompanying notes/slides

discussion items

- course website http://pillowlab.princeton.edu/teaching/mathtools16fall/
- sign up for piazza.
- take math-background poll

- for Python newbies:
 - check out "Python Starter Pack" (on piazza) and install Anaconda / Python

- Today: **"Intro to linear algebra: vectors and some of the basic stuff you can do with them."**

today's topics

- vectors (geometric picture)
    - vector addition
    - scalar multiplication
- vector norm ("L2 norm")
- unit vectors
- dot product ("inner product")
- linear projection
- orthogonality

vectors

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$
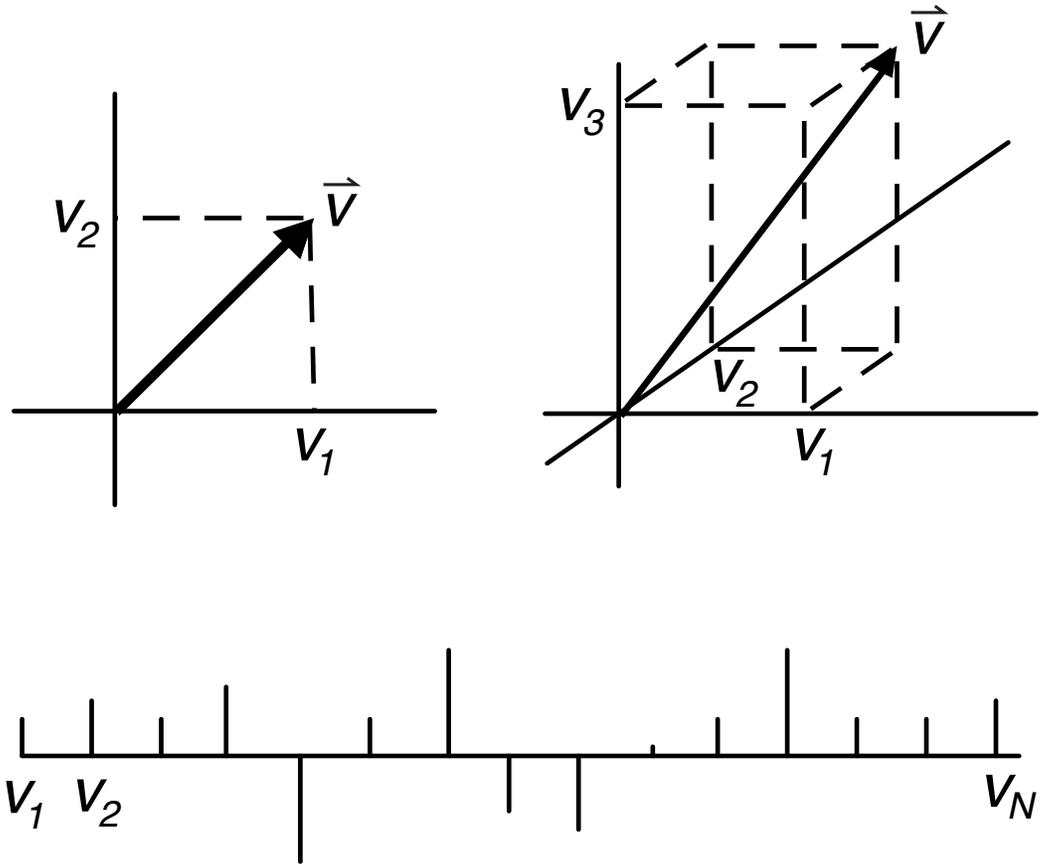
column vector

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$

in python

```
# make a 3-component column vector
v = np.array([[3], [1], [-7]])
```
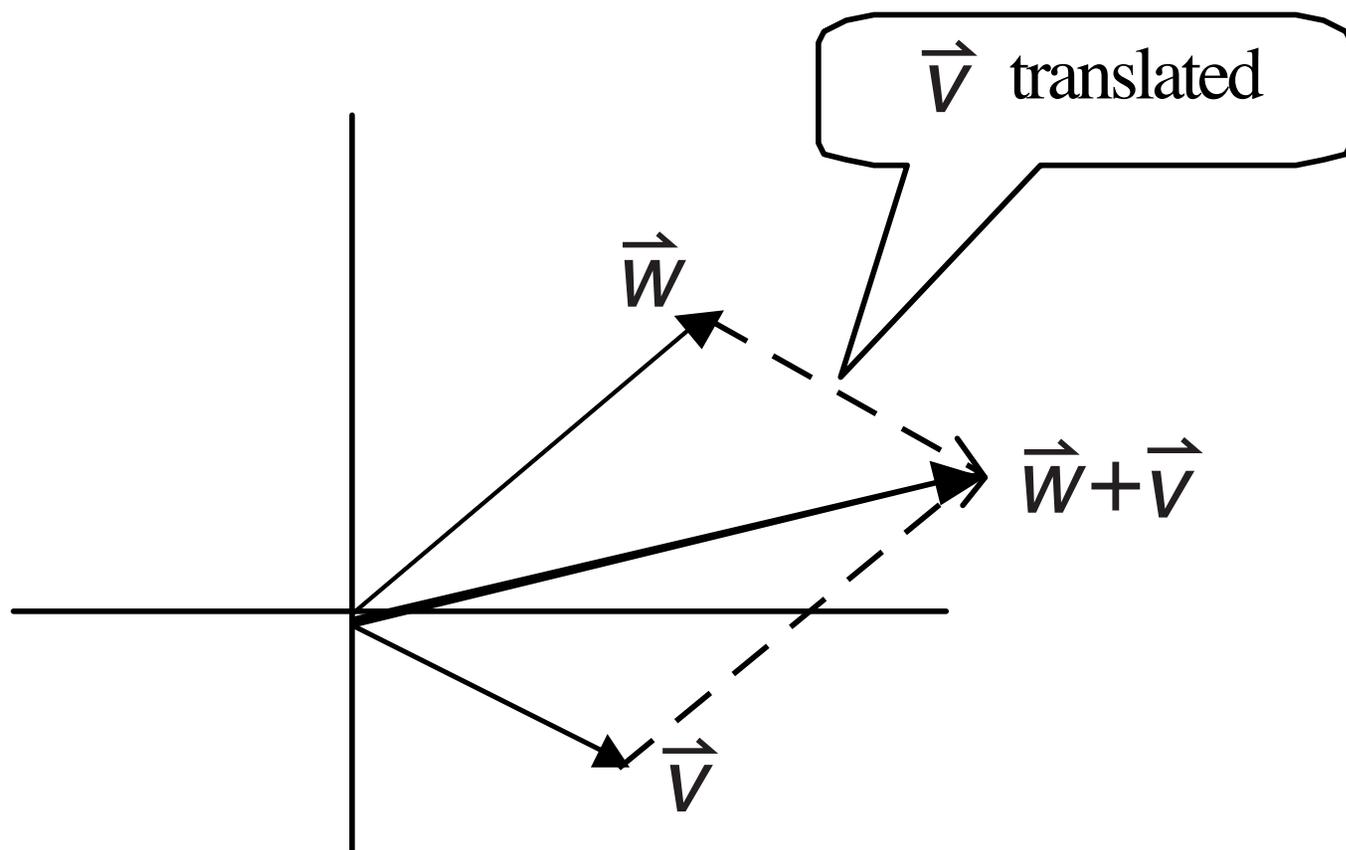
transpose

```
# transpose
v.T
```

$$\vec{v}^T = \begin{pmatrix} v_1 & v_2 & \cdots & v_N \end{pmatrix}$$

row vector

```
# create row vector directly
v = np.array([[3,1,-7]]) # row vector
# or
v = np.array([3,1,-7]) # 1D vector
```
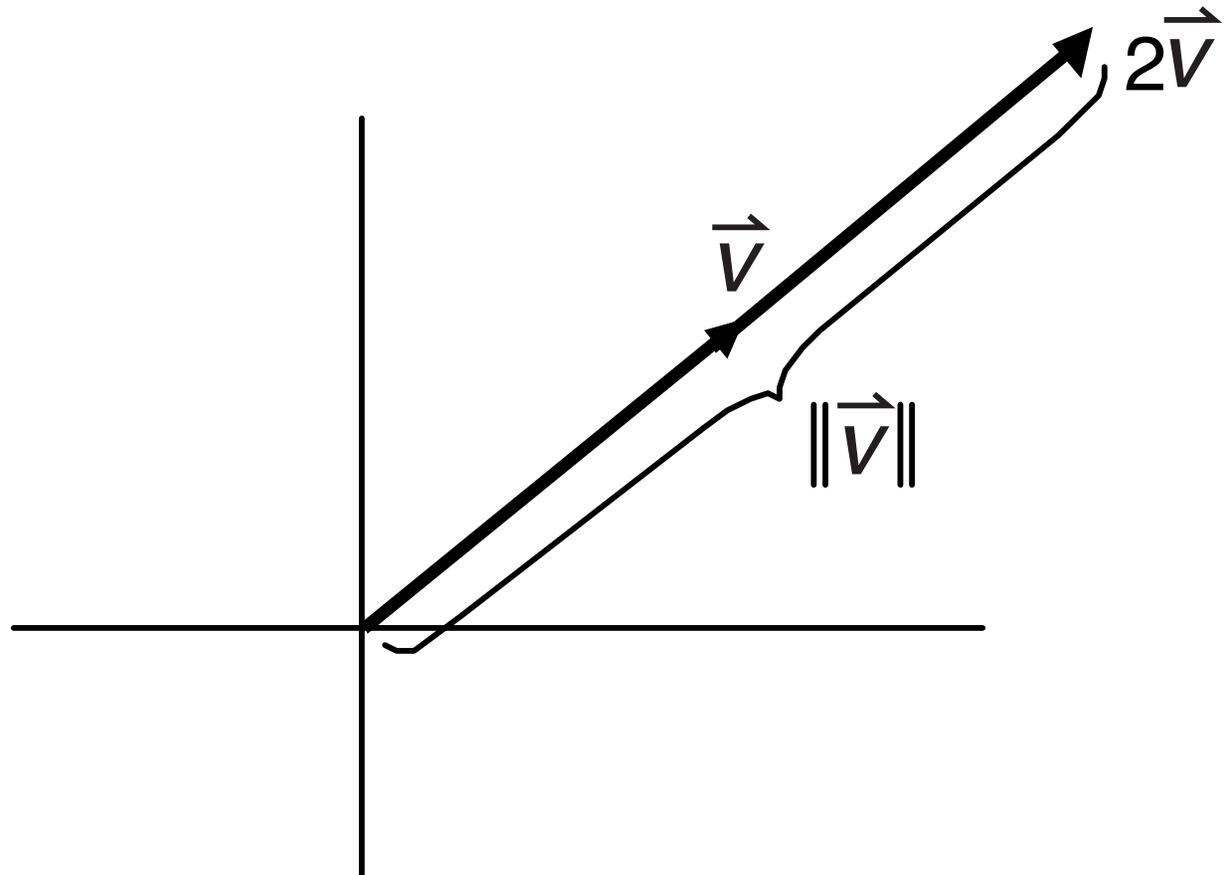
addition of vectors

$$\vec{v} + \vec{w} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \end{pmatrix}$$

scalar multiplication

$$a\vec{v} = a \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 \\ av_2 \end{pmatrix}$$
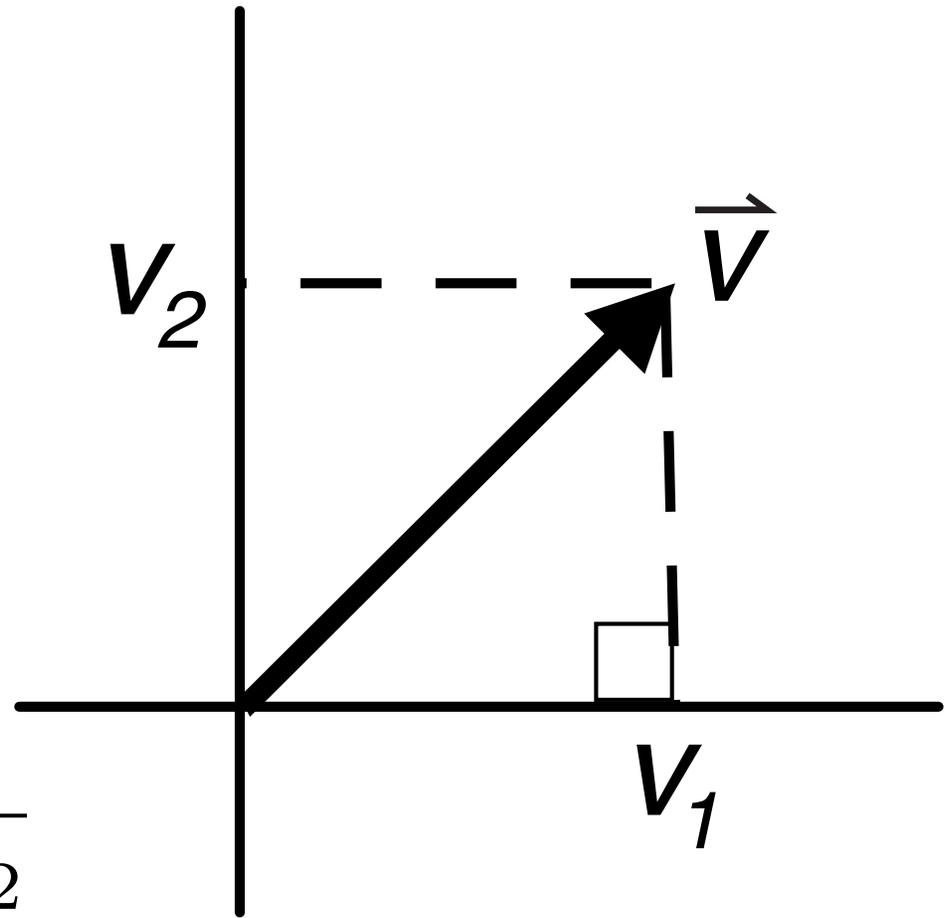
vector norm ("L2 norm")

- vector length in Euclidean space

In 2-D:

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$$

In *n*-D:

$$\|\vec{v}\| = \sqrt{v_1^2 + \cdots + v_n.^2}$$

vector norm ("L2 norm")
in python

```python
# make a vector
v = np.array([1, 7, 3, 0, 1])

# many equivalent ways to compute norm
np.linalg.norm(v)        # built-in function
np.sqrt(np.dot(v,v))     # sqrt of dot product
np.sqrt(v.T @ v)         # sqrt of v-tranpose times v
np.sqrt(sum(v * v))      # sqrt of sum of elementwise product
np.sqrt(sum(v ** 2))     # sqrt of v elementwise-squared

# note use of @ and * and **
# @ - gives matrix multiply
# * - gives elementwise multiply
# ** - gives exponentiation ("raising to a power")
```
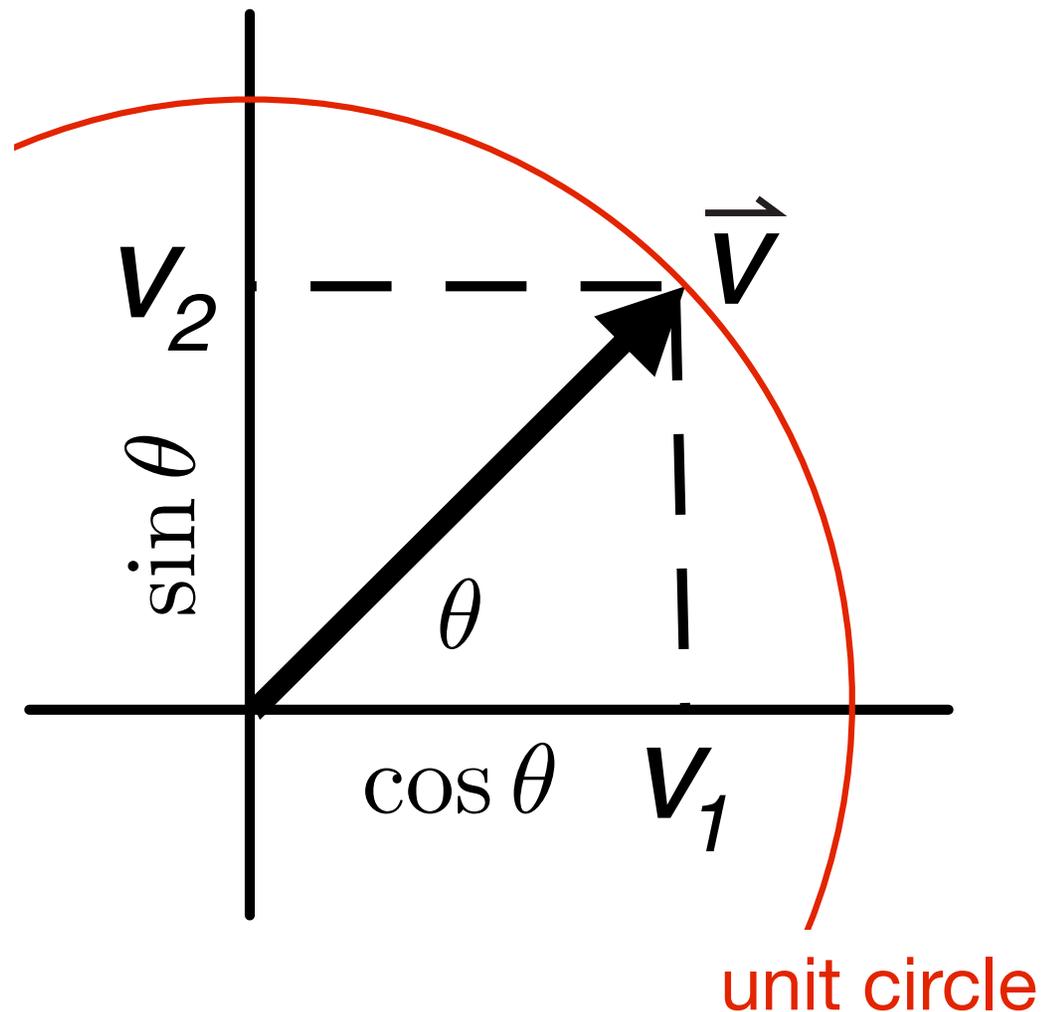
unit vector

- vector such that $||\vec{v}|| = 1$

- <u>in 2 dimensions</u>

$$\vec{v} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$
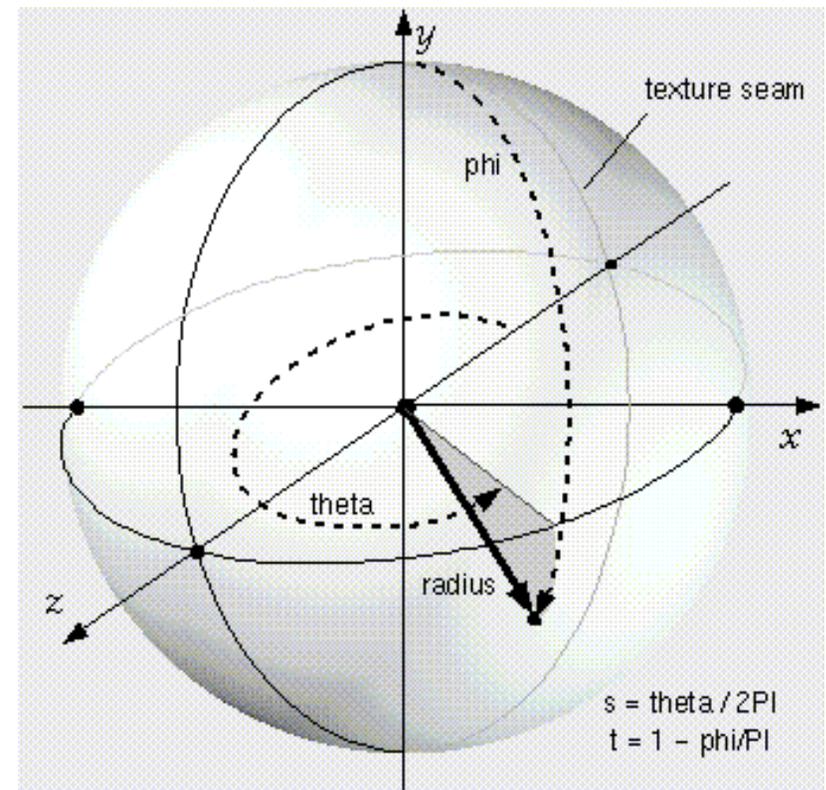
$$\cos^2\theta + \sin^2\theta = 1$$

unit vector

- vector such that $\|\vec{v}\| = 1$

- in *n* dimensions
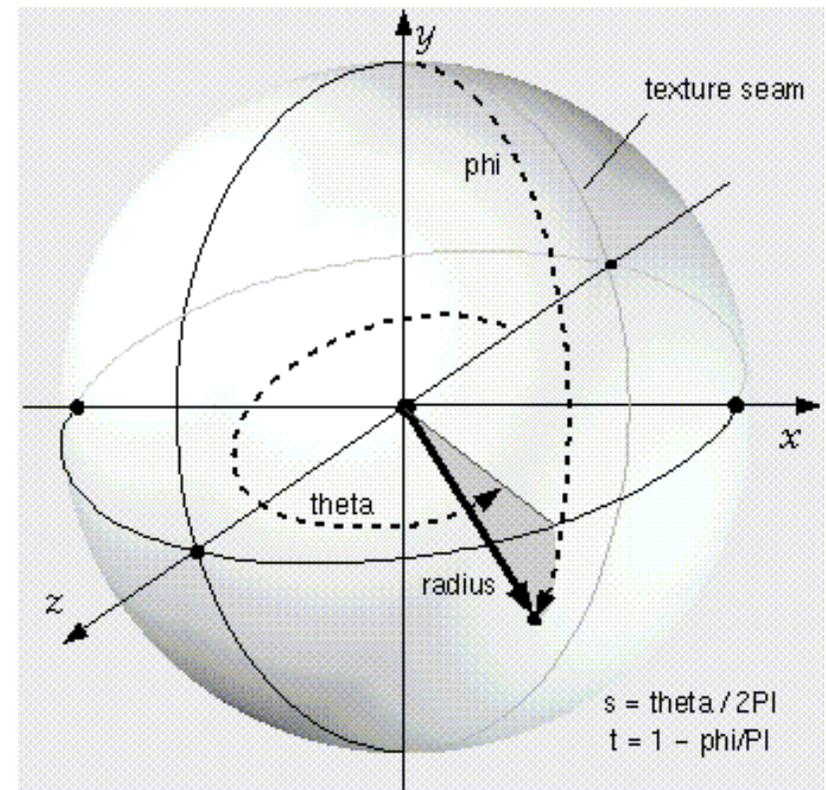
$$v_1^2 + v_2^2 + \ldots + v_n^2 = 1$$

- sits on the surface of an *n*-dimensional hypersphere

unit vector

- vector such that $\|\vec{v}\| = 1$

- make any vector into a unit vector via

$$\frac{1}{\|\vec{v}\|}\vec{v}$$

inner product (aka "dot product")
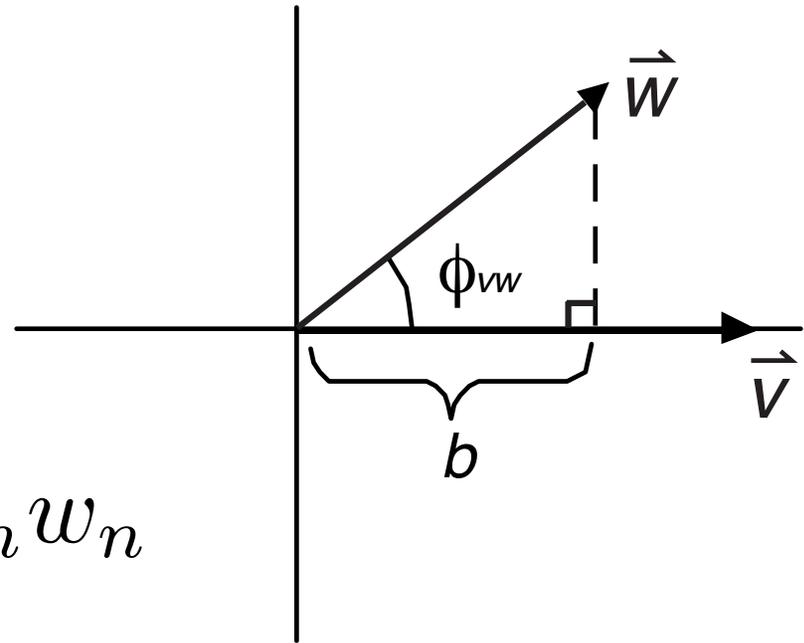
- produces a scalar from two vectors

$$\vec{v} \cdot \vec{w}$$

$$\langle \vec{v}, \vec{w} \rangle$$

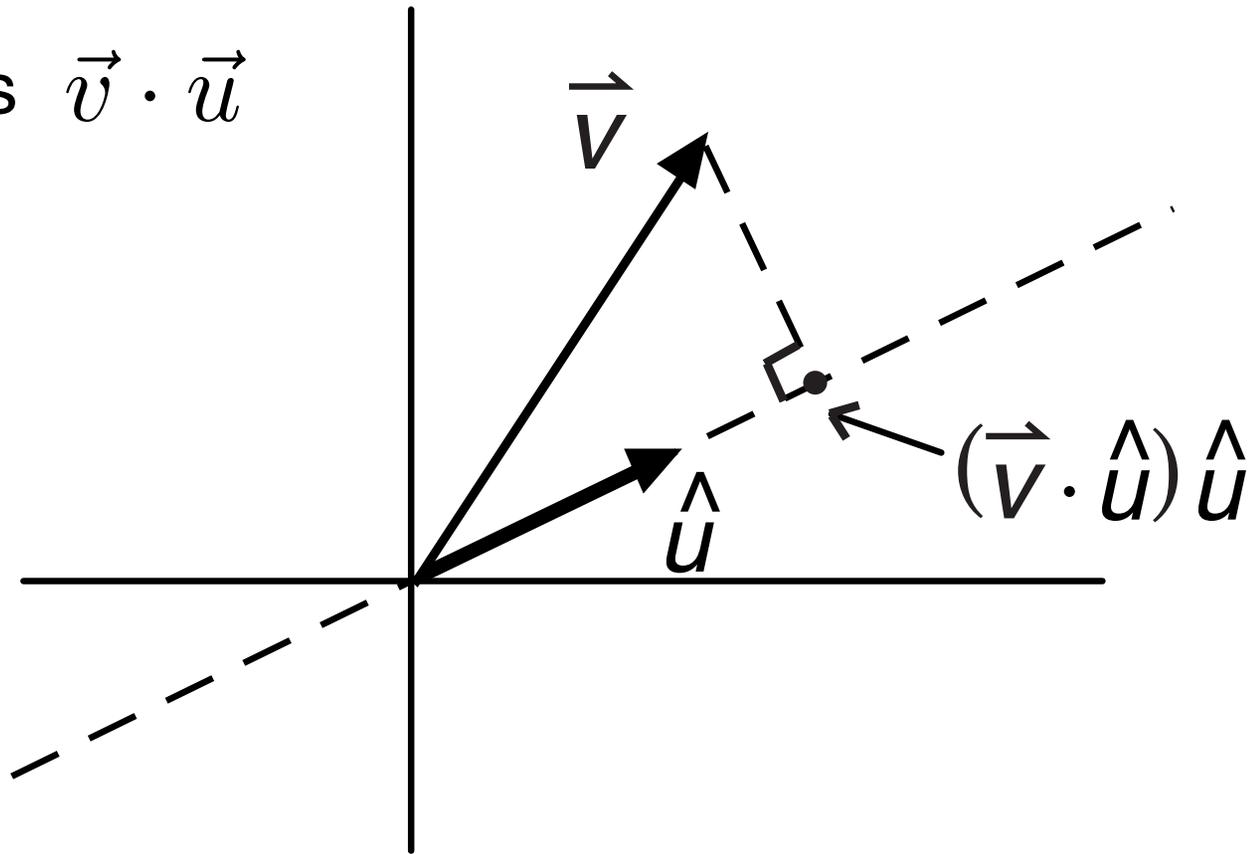$$v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$

$$\|v\| \|w\| \cos \theta$$

$$\vec{v}^T \vec{w} = \begin{pmatrix} v_1 & \cdots & v_n \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$

linear projection

- intuitively, dropping a vector down onto a linear surface at a right angle

- if *u* is a unit vector,
  length of projection is $\vec{v} \cdot \vec{u}$

$$\vec{v}$$

$$\hat{u}$$

$$(\vec{v} \cdot \hat{u}) \, \hat{u}$$

- for non-unit vector, length of projection = $\vec{v} \cdot \left(\frac{1}{||\vec{u}||} \vec{u} \right)$
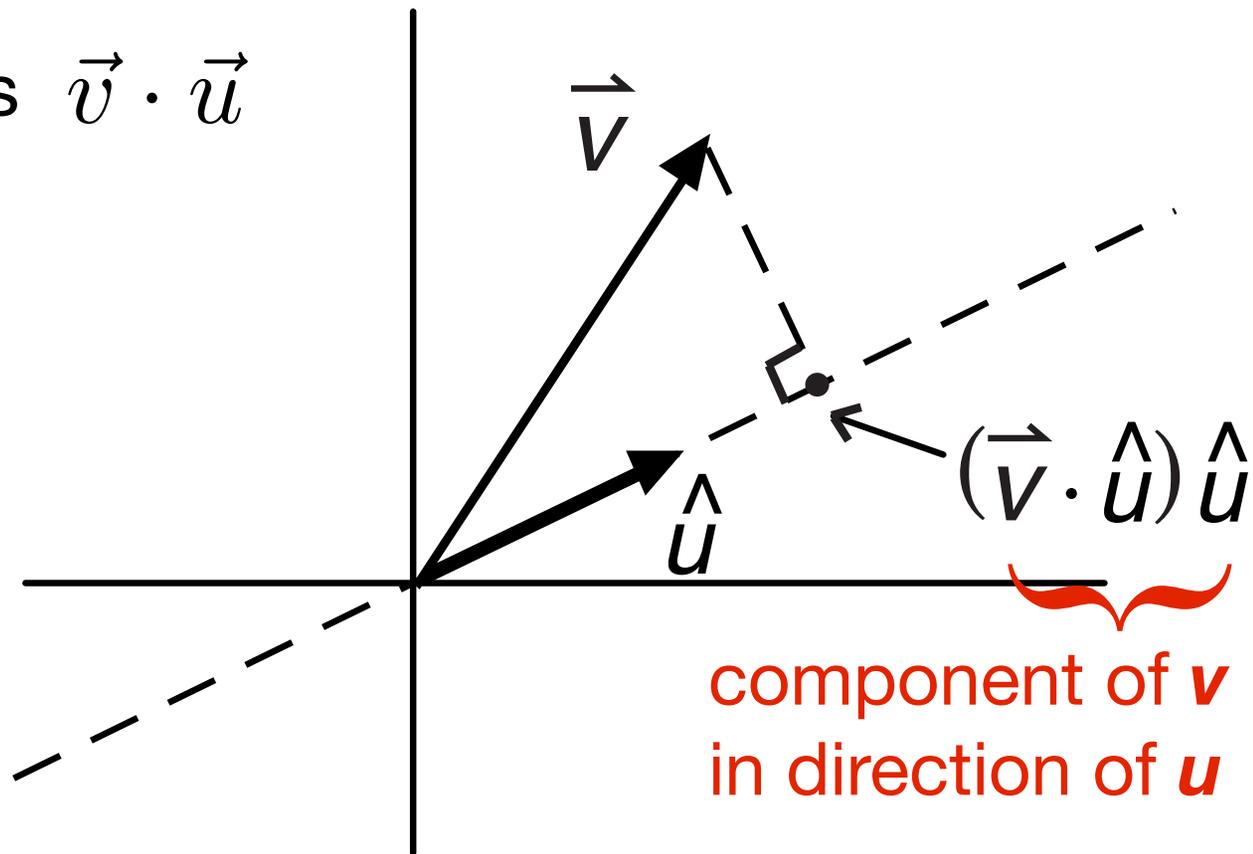
linear projection

- intuitively, dropping a vector down onto a linear surface at a right angle

- if u is a unit vector,
  length of projection is $\vec{v} \cdot \vec{u}$

$$(\vec{v} \cdot \hat{u})\, \hat{u}$$

component of **v**
in direction of **u**

- for non-unit vector, length of projection = $\vec{v} \cdot \left(\frac{1}{||\vec{u}||}\, \vec{u} \right)$

orthogonality

- two vectors are orthogonal (or "perpendicular") if their dot product is zero: $\vec{v} \cdot \vec{w} = 0$

component of **v** orthogonal to **u**

$$\vec{v} - (\vec{v} \cdot \vec{u})\vec{u}$$

$\vec{v}$

$\hat{u}$

$(\vec{v} \cdot \hat{u})\hat{u}$

component of **v** in direction of **u**