Linear Algebra II: vector spaces

Math Tools for Neuroscience (NEU 314)
Spring 2016

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Lecture 3
(Tuesday 2/9)

accompanying notes/slides

discussion items

- Up now on piazza:
  - Chapter 2 of *Wallisch et al* (Matlab for Neuroscientistis)
  - homework 0 (Matlab basics).

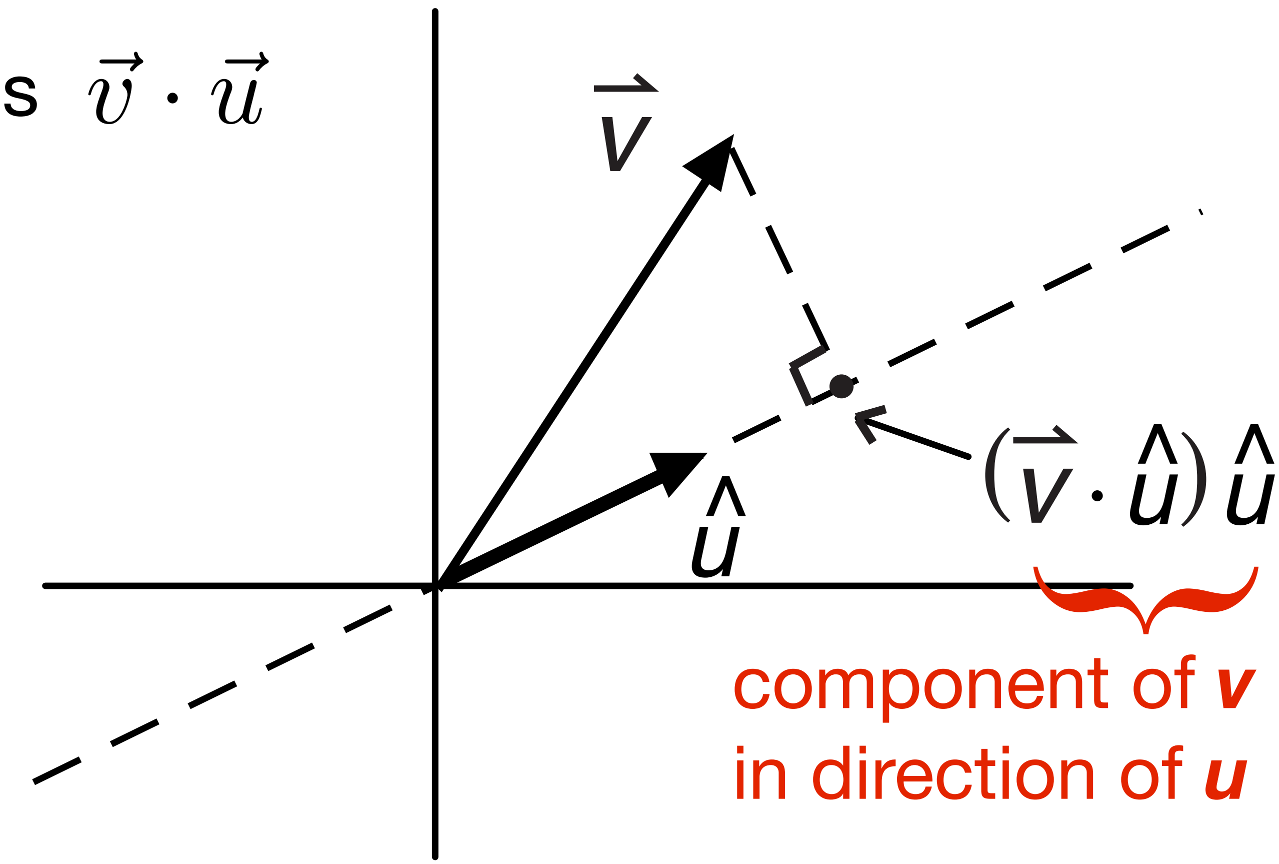
- This week in lab: do interactive lab ("lab 1" linked from website) and then work on homework 0.

today's topics

- linear projection
- orthogonality
- linear combination
- linear independence / dependence
- vector space
- subspace
- basis
- orthonormal basis

linear projection

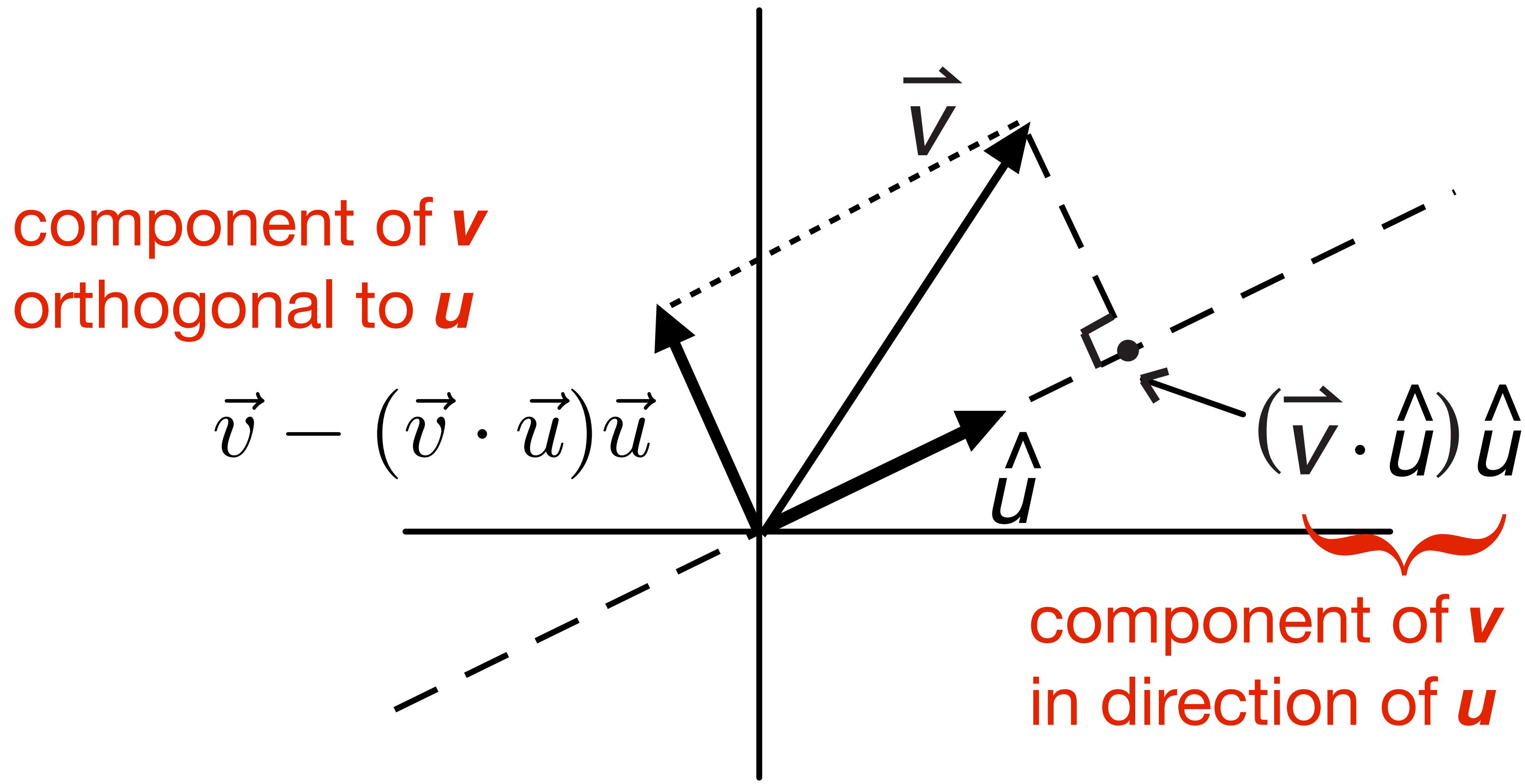
- intuitively, dropping a vector down onto a linear surface at a right angle
- if *u* is a unit vector, length of projection is $\vec{v} \cdot \vec{u}$



- for non-unit vector, length of projection = $\vec{v} \cdot \left(\frac{1}{||\vec{u}||} \vec{u} \right)$

orthogonality

- two vectors are orthogonal (or "perpendicular") if their dot product is zero: $\vec{v} \cdot \vec{w} = 0$

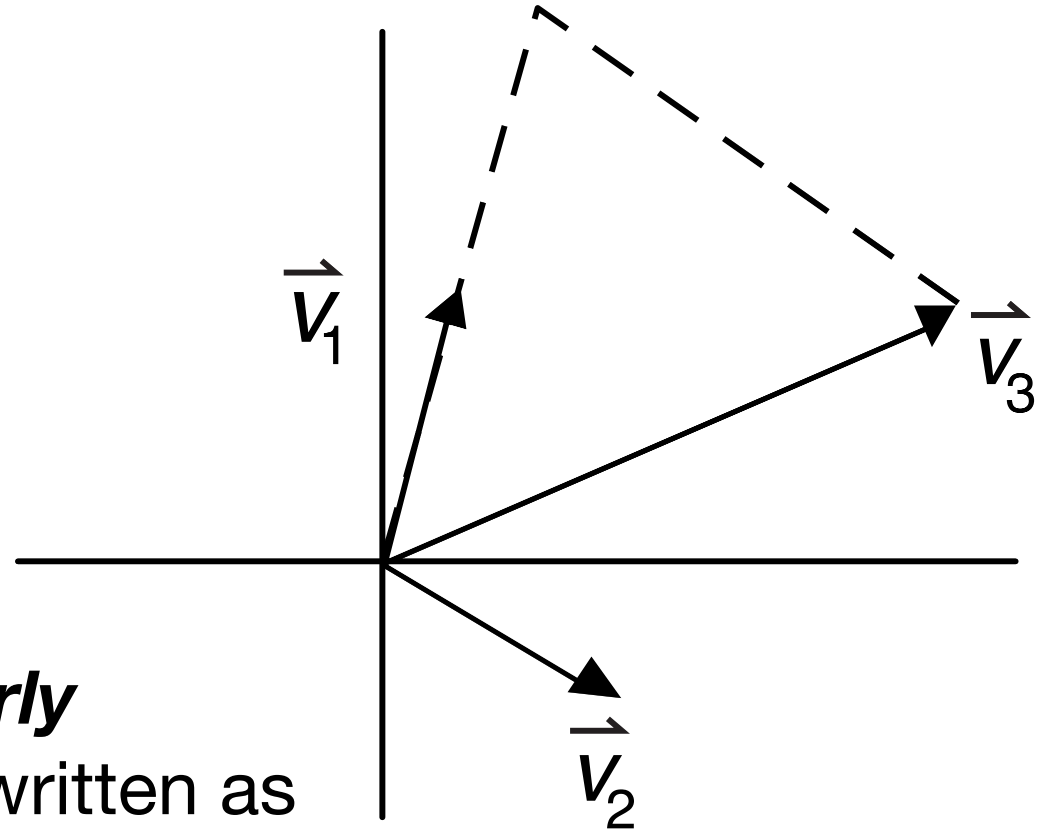


- Can decompose any vector into its component along u and its residual (orthogonal) component.

linear combination

- scaling and summing applied to a group of vectors

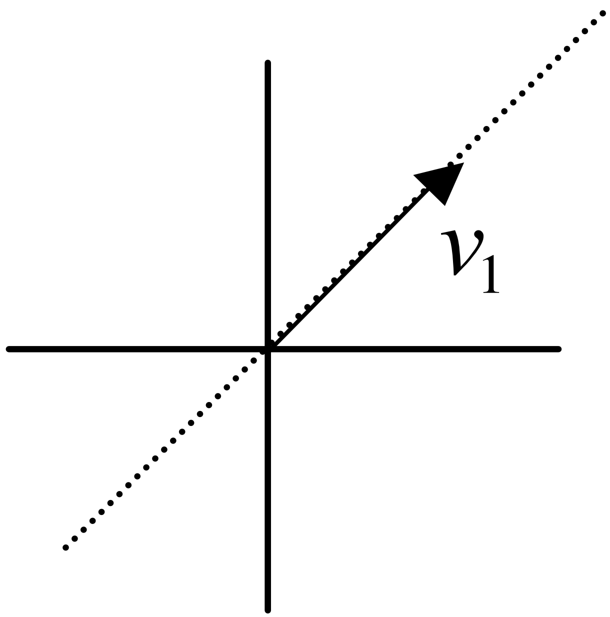
$$a\vec{v}_1 + b\vec{v}_2 = \vec{v}_3$$



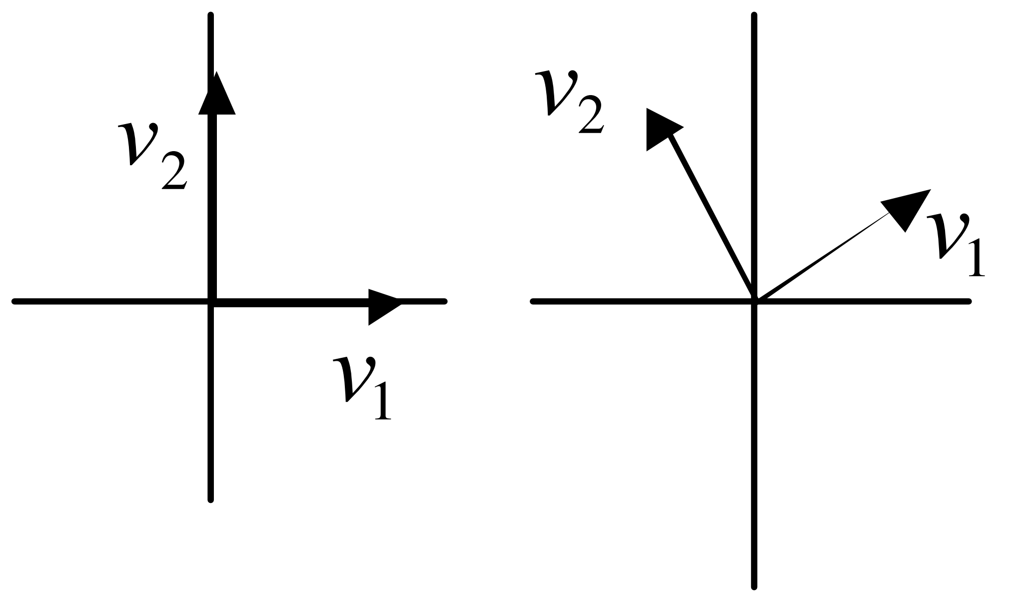
- a group of vectors is ***linearly dependent*** if one can be written as a linear combination of the others
- otherwise, ***linearly independent***

vector space

- set of all points that can be obtained by linear combinations of some set of “basis” vectors



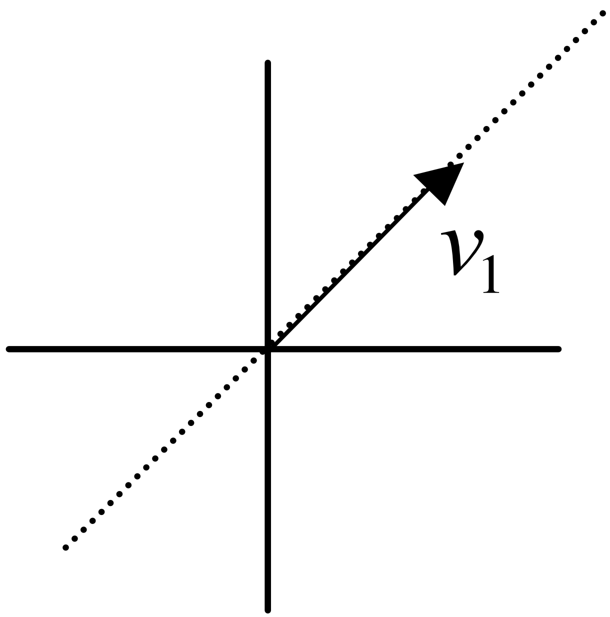
1D vector space spanned by single basis vector



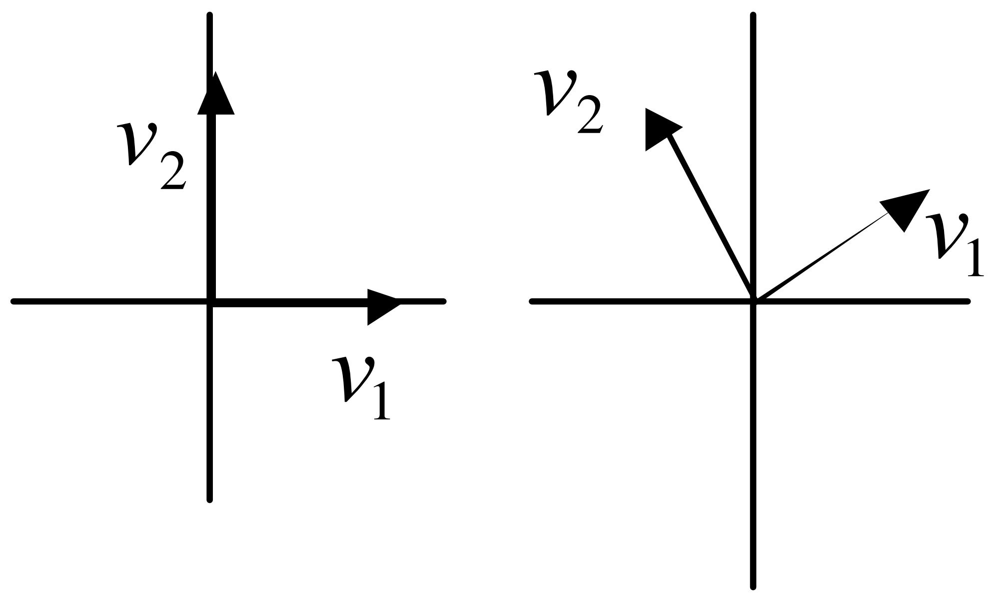
Two different (orthonormal) bases for the same 2D vector space

vector space

- set of all points that can be obtained by linear combinations of some set of “basis” vectors



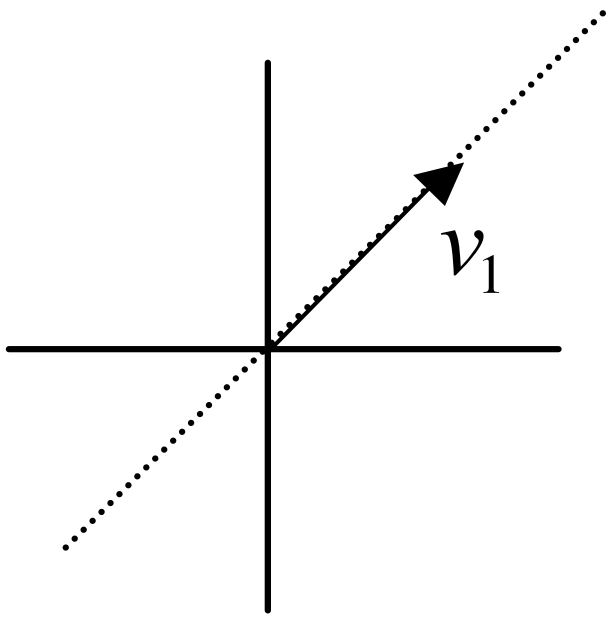
1D vector space
(***subspace*** *of R^2*)



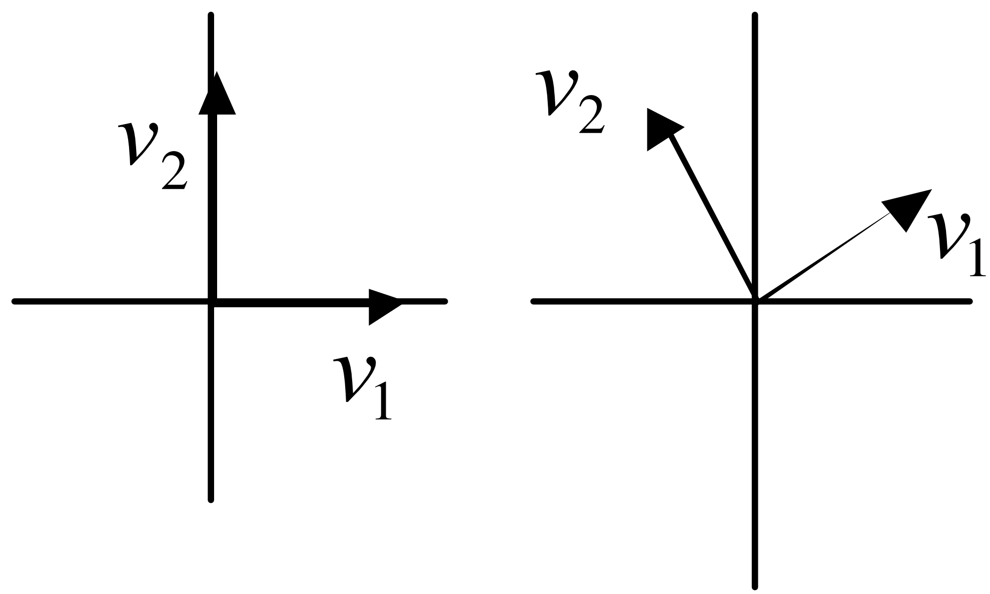
Two different (orthonormal) bases for the same 2D vector space

basis

- set of vectors that can “**span**” (form via linear combination) all points in a vector space



1D vector space (***subspace*** *of* R^2)



Two different (orthonormal) bases for the same 2D vector space

orthonormal basis

- basis composed of orthogonal unit vectors