Math Tools for Neuroscience (NEU 314)
Spring 2016

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accompanying notes/slides for Thursday, Feb 4

discussion items

- course website: http://pillowlab.princeton.edu/teaching/mathtools16/
- sign up for piazza.
- take math-background poll

- for Matlab newbies:
  - check out MATLAB for neuroscientists (Lewis Library), chapter 2
  - either of the matlab tutorials posted on course website (just first part, up through vectors & matrices).

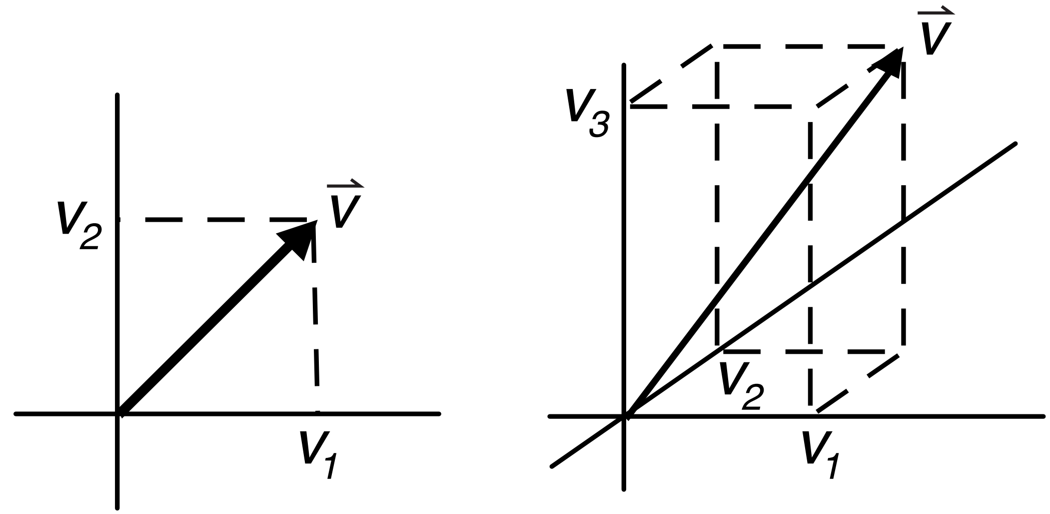
- Today: "**Intro to linear algebra: vectors and some of the basic stuff you can do with 'em.**"
- Next week: start of labs!

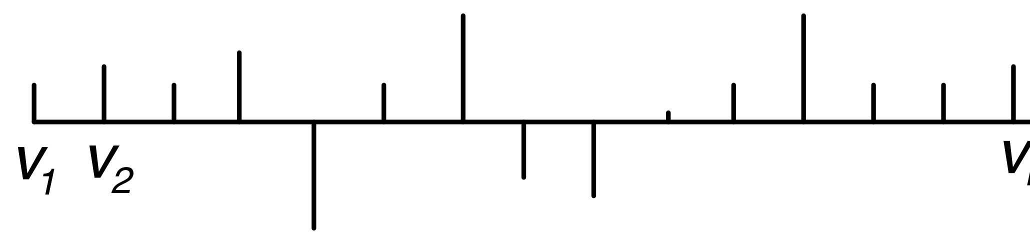
today’s topics

- vectors (geometric picture)
    - vector addition
    - scalar multiplication
- vector norm (“L2 norm”)
- unit vectors
- dot product (“inner product”)
- linear projection
- orthogonality

vectors

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$





column vector

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{pmatrix}$$

in matlab

```
% make a 5-component
% column vector

v = [1; 7; 3; 0; 1];
```

transpose

$$\vec{v}^T = \begin{pmatrix} v_1 & v_2 & \cdots & v_N \end{pmatrix}$$

row vector

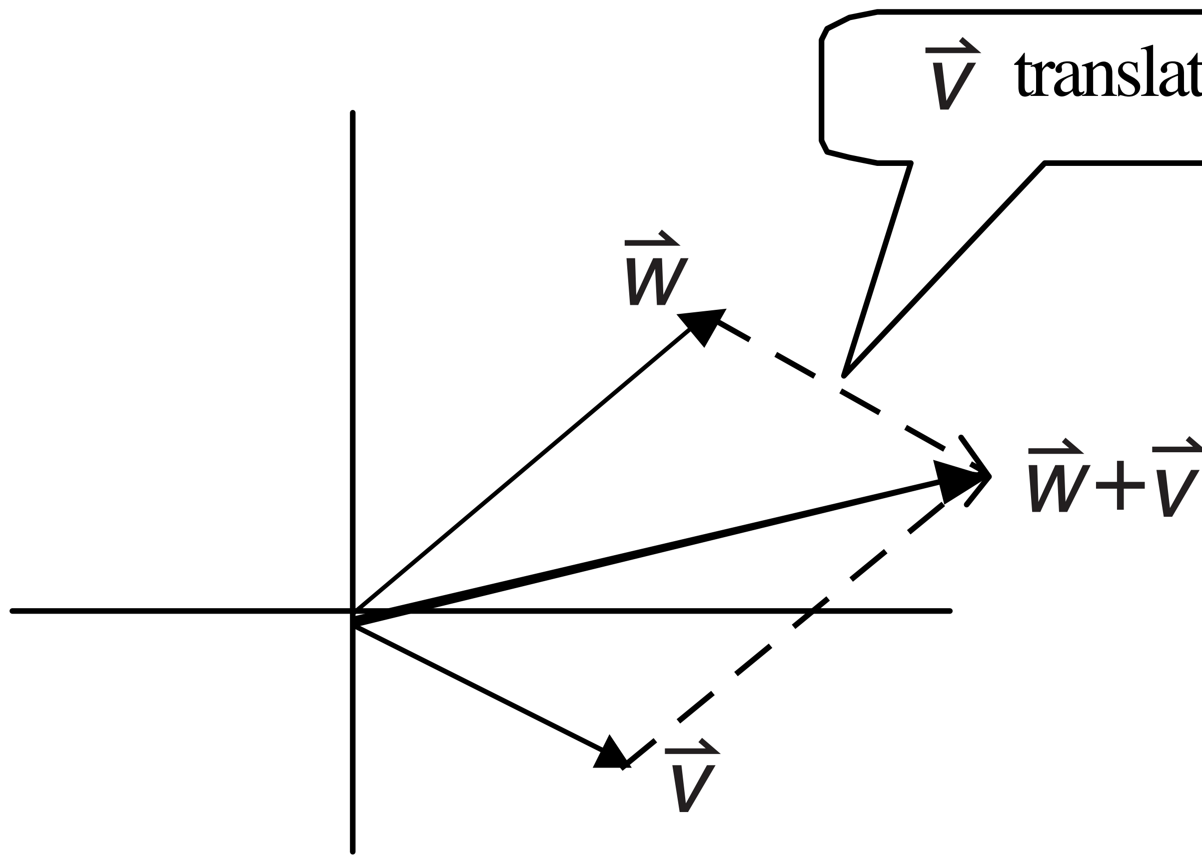
```
% transpose
v'
```

```
% create row vector by
% separating with ,
% instead of ;

v = [1, 7, 3, 0, 1];
```

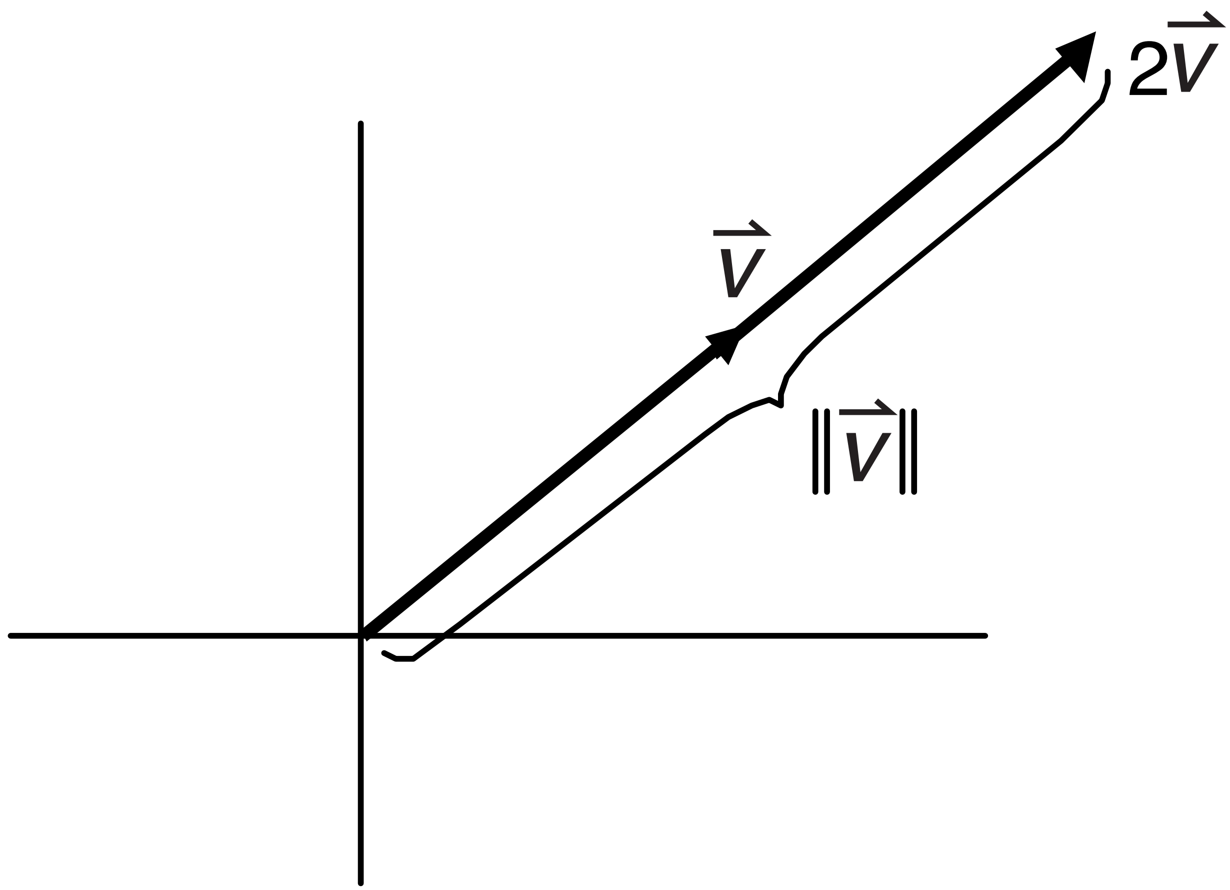
addition of vectors

$$\vec{v} + \vec{w} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \end{pmatrix}$$



scalar multiplication

$$a\vec{v} = a\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} av_1 \\ av_2 \end{pmatrix}$$



vector norm (“L2 norm”)

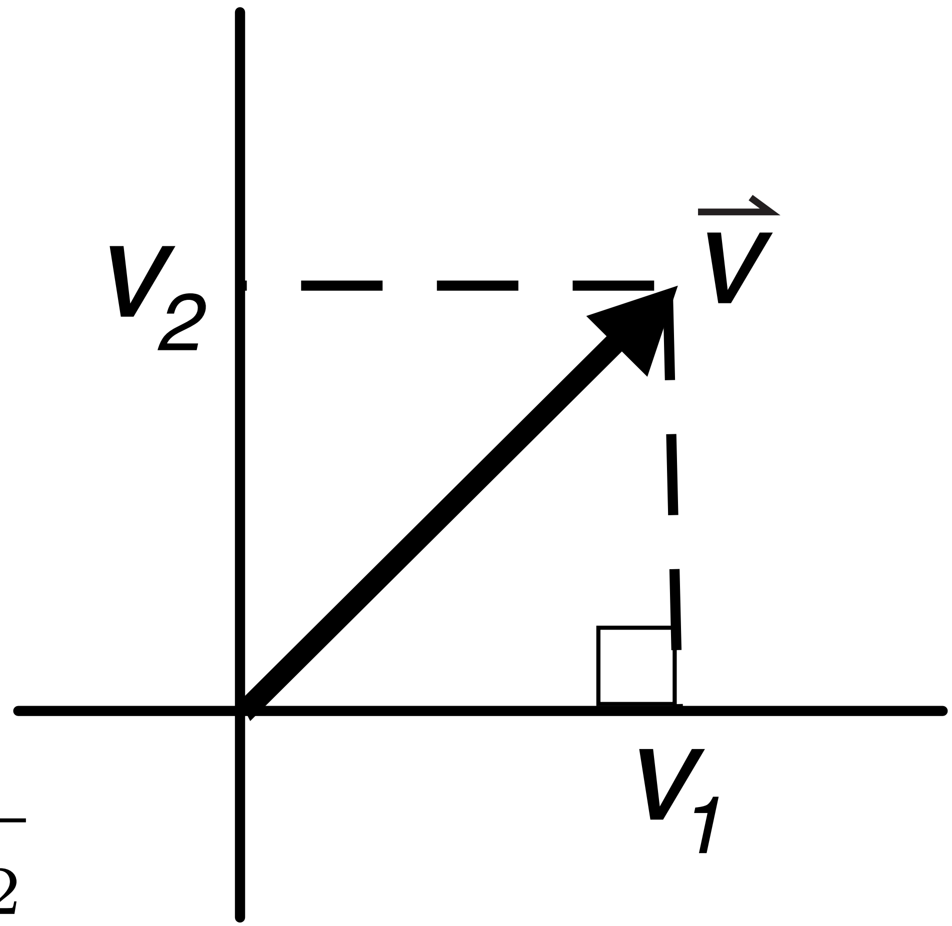
- vector length in Euclidean space

<u>In 2-D:</u>

$$||\vec{v}|| = \sqrt{v_1^2 + v_2^2}$$

<u>In *n*-D:</u>

$$||\vec{v}|| = \sqrt{v_1^2 + \cdots + v_n.^2}$$



vector norm (“L2 norm”)

in matlab

```
v = [1; 7; 3; 0; 1]; % make a vector

% many equivalent ways to compute norm

norm(v)
sqrt(v'*v)
sqrt(v(:).^2)
sqrt(v(:).*v(:))


% note use of .* and .^, which operate
% 'elementwise' on a matrix or vector
```

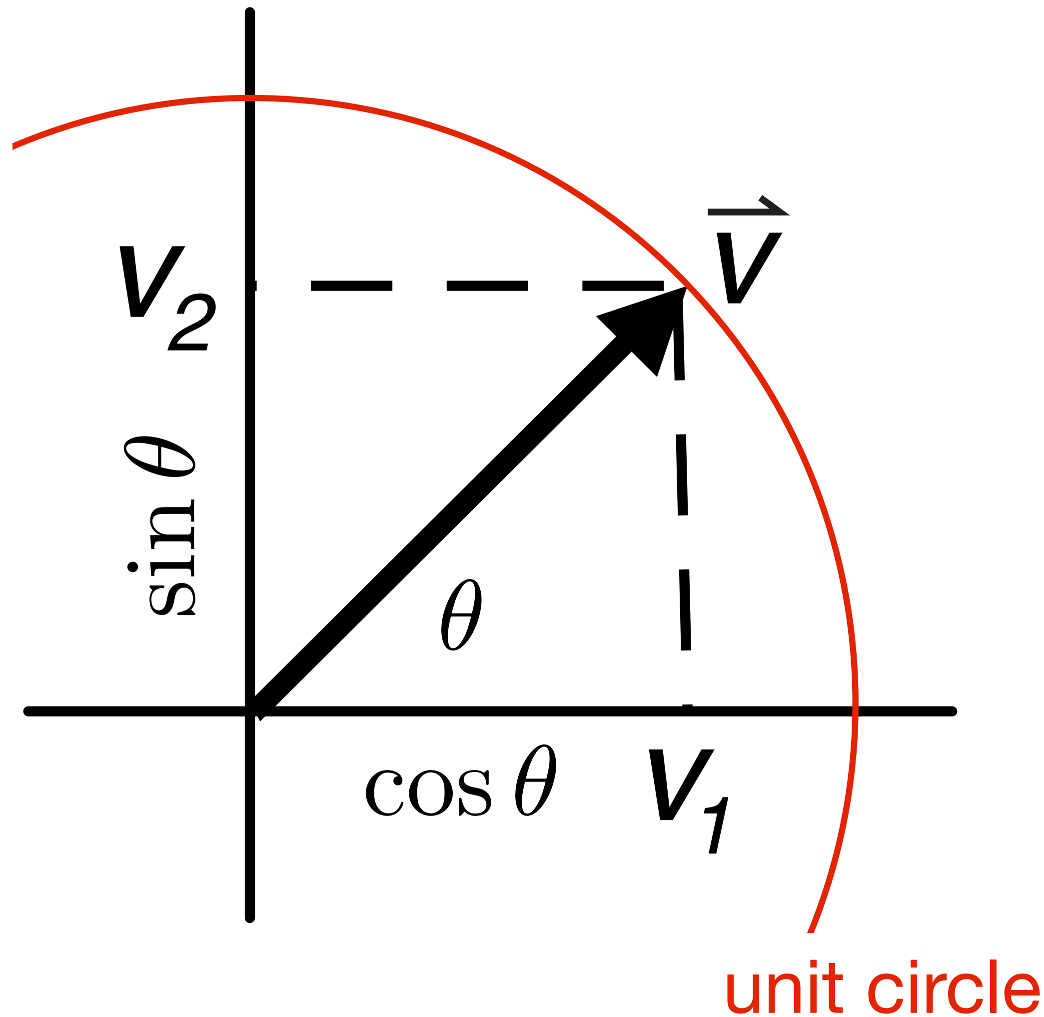
unit vector

- vector such that $||\vec{v}|| = 1$

- <u>in 2 dimensions</u>

$$\vec{v} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

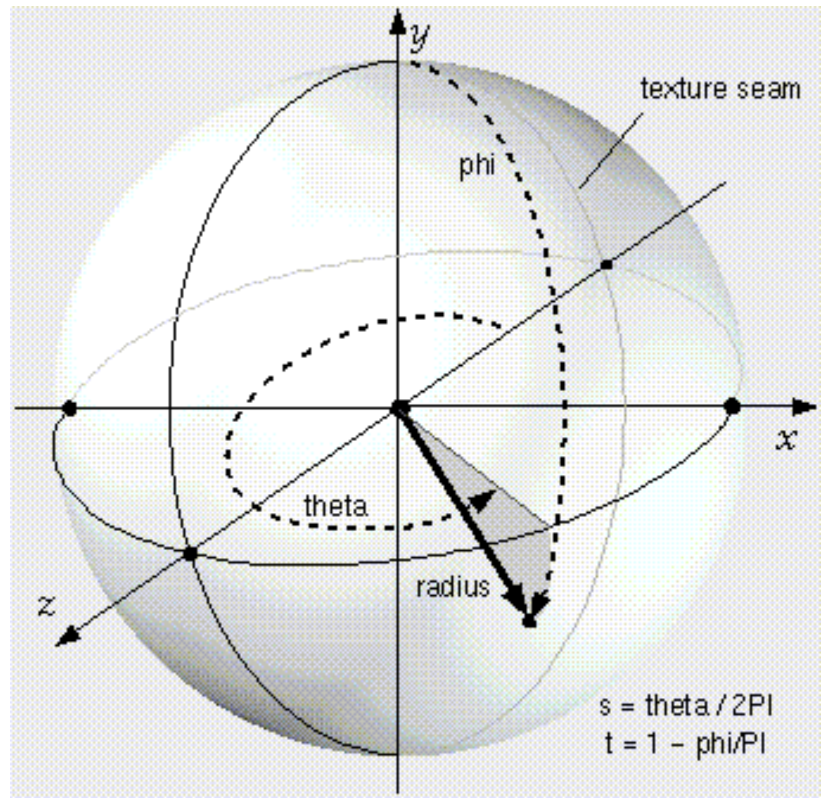
$$\cos^2\theta + \sin^2\theta = 1$$



unit vector

- vector such that $||\vec{v}|| = 1$

- <u>in *n* dimensions</u>



$$v_1^2 + v_2^2 + \ldots + v_n^2 = 1$$

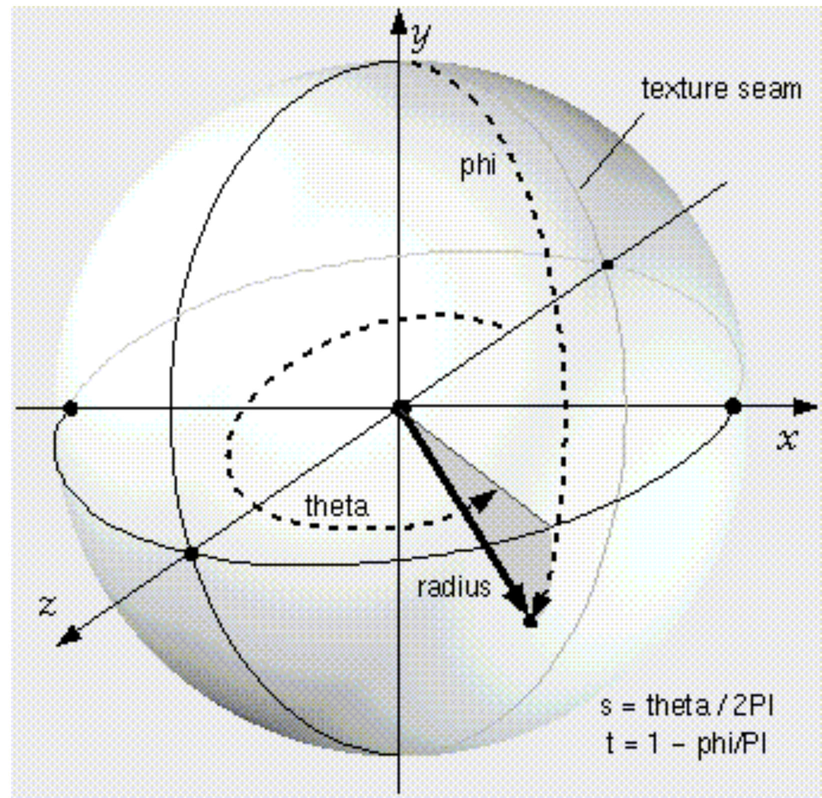
- sits on the surface of an *n*-dimensional hypersphere

unit vector

- vector such that $||\vec{v}|| = 1$

- make any vector into a unit vector via

$$\frac{1}{||\vec{v}||}\vec{v}$$



inner product (aka "dot product")

- produces a scalar from two vectors

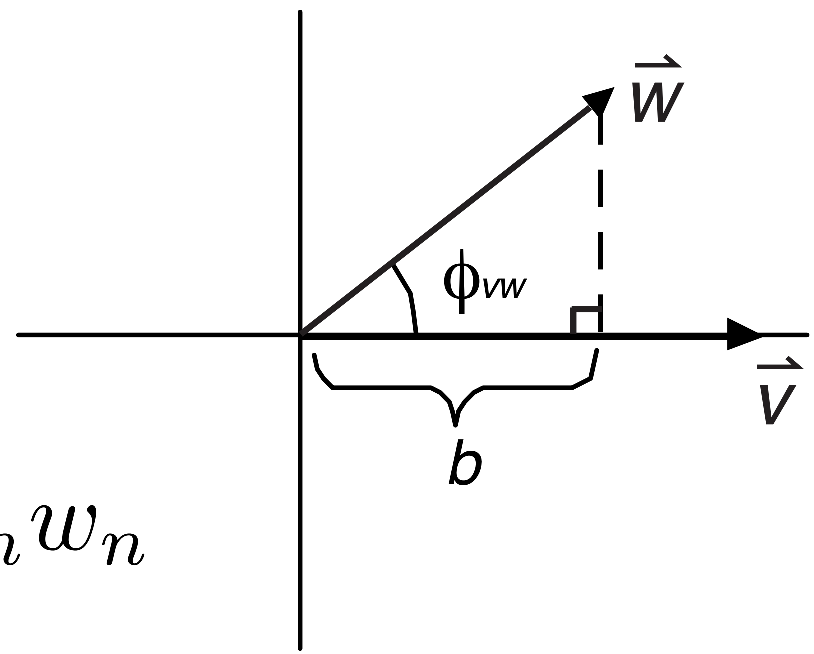
$$\vec{v} \cdot \vec{w}$$

$$\langle \vec{v}, \vec{w} \rangle$$

$$v_1 w_1 + v_2 w_2 + \cdots + v_n w_n$$

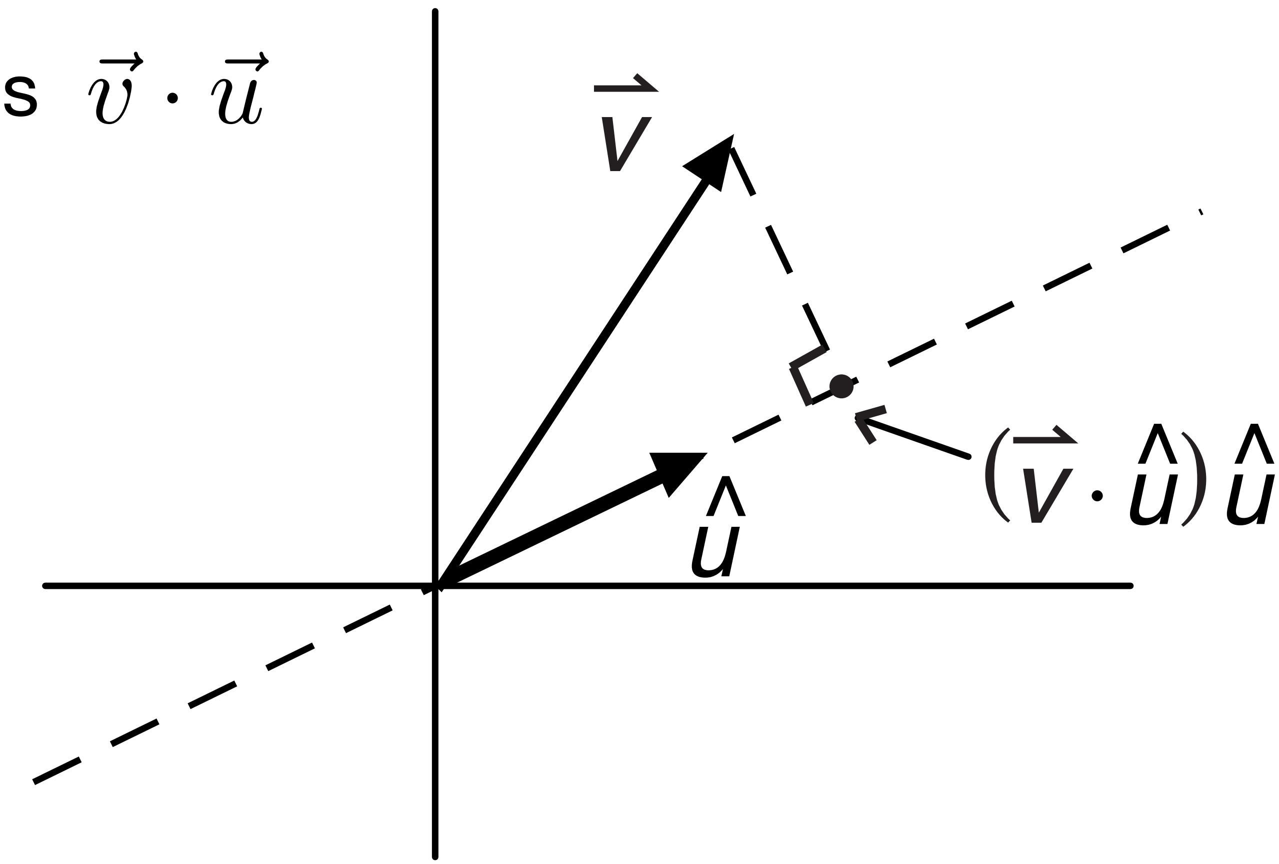
$$\|v\| \|w\| \cos\theta$$

$$\vec{v}^T \vec{w} = \begin{pmatrix} v_1 & \cdots & v_n \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$$



linear projection

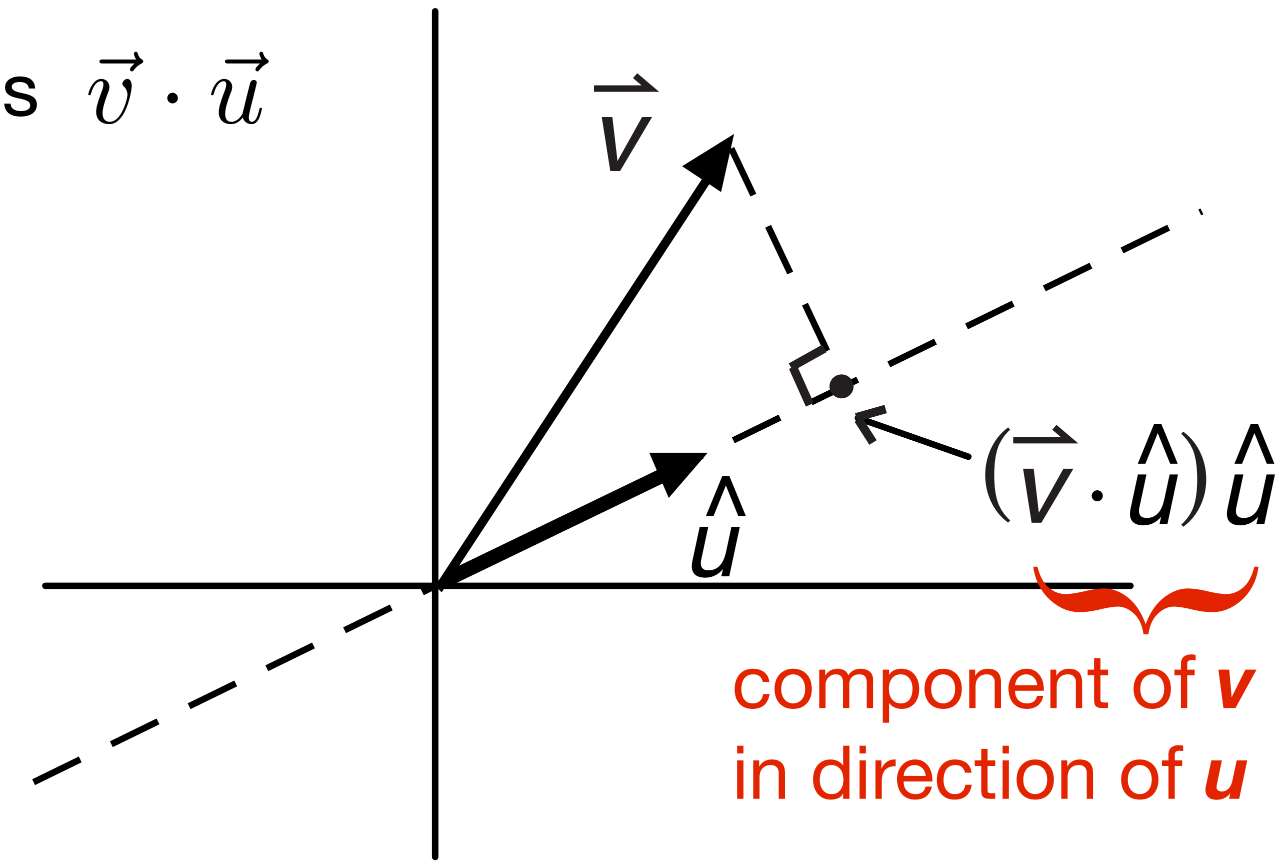
- intuitively, dropping a vector down onto a linear surface at a right angle
- if *u* is a unit vector, length of projection is $\vec{v} \cdot \vec{u}$



- for non-unit vector, length of projection = $\vec{v} \cdot \left(\frac{1}{||\vec{u}||} \vec{u} \right)$

linear projection

- intuitively, dropping a vector down onto a linear surface at a right angle
- if *u* is a unit vector, length of projection is $\vec{v} \cdot \vec{u}$



- for non-unit vector, length of projection = $\vec{v} \cdot \left(\frac{1}{||\vec{u}||} \vec{u} \right)$

orthogonality

- two vectors are orthogonal (or "perpendicular") if their dot product is zero: $\vec{v} \cdot \vec{w} = 0$