Mathematical Tools for Neuroscience (NEU 314)
Princeton University, Spring 2016
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Homework 2: more linear algebra and color vision

Due: Tuesday, Mar 1, 9:59am

Formatting Instructions:

- Please prepare your homework submission as a single script (m-file), with %% separating each problem or sub-problem, so that each can be executed with a `ctrl-enter`. See the posted solution for homework 0 to see an example of the desired formatting.
- Use comments (lines preceded by `%`) to respond to any questions requiring additional explanation.
- Several problems will require writing a separate function (which will be called from your script). Place the main script and any necessary functions in a folder called `lastname_hw2`. Zip this folder and email it to the TA for your section by the due date.
- Problems or lettered sub-problems are worth 1 point each, unless otherwise specified.
- Late submissions will be penalized one letter grade (10%) per day.

Linear Systems: Row Space and Null Space

1. **A simple linear system.** Suppose you have a retinal neuron whose response is a weighted sum of the intensities of light that land on 6 photoreceptors (note that these intensities are positive values). The weight vector is $[3, 1, 5, 4, 2, 1]$.

   a. What unit-length stimulus vector elicits the largest response in the neuron? (Explain how you arrived at your answer.)

   b. Now generate a unit-vector stimulus vector that elicits a zero response in the neuron (and verify that this is true).

   c. Is this a physically realizable stimulus? Is there *any* realizable stimulus (not necessarily unit length) that would elicit a zero response in the neuron? If so, give an example.

2. **Null space**. The null space of a matrix M is the vector space consisting of all vectors $\vec{x}$ for which $M\vec{x} = 0$. In other words, it is the subspace that contains all vectors that are orthogonal to the vectors in the *row space* of M (i.e., the vector space spanned by the rows of M).

   Load the file `mtx`Examples.mat into your MATLAB world. You'll find a set of matrices named `mtx`N, with $N = 1, ..., 5$. For each matrix:

   a. determine if there are vectors in the input space that the matrix maps to zero (i.e., determine if there is a null space), and, if so, *b*. construct an orthonormal basis for the null space, and

c. generate a random vector that lies in the null space, and verify that the matrix maps it to the zero vector.

(1 point for each matrix).

3. Now, write a function `findNullSpace.m` that takes in an $m \times n$ matrix and returns an orthonormal basis for its null space. (5 points).

   Notes:

   • if you like, you can do #3 first and use the function you created for tackling #2 above. Alternatively, you can work out the specific solutions in #2 and then use these intuitions to write the general function. (Hint: you can use an algorithm similar to the one you used for Gram-Schmidt orthogonalization).

   • There is a matlab function `null` that will find the null space of a matrix: don't use this in any of your solutions above (but you might to use it to check that the answer you're getting makes sense).

More Color Vision

Load the file `colrmatch.mat` that you used for the first homework problem. You'll need the variable `Phosphors` (an $N \times 3$ matrix containing the illuminant power spectrum of three standard color monitor phosphors) and the variable `Cones`, which contains spectral sensitivities of the three cones in the human eye (ordered such that the first row, `Cones(1,:)`, corresponds to the L (red) cone, and the second and third rows correspond to M (green) and S (blue) cones, respectively). The vector of wavelengths is given in the variable `wavelengths`.

4. **Color-blindness (dichromacy)**

   a. Write a function `isVisibleForProtanope.m` that determines whether two colors presented on a computer monitor are distinguishable to a dichromat missing the L cone. The function takes in `Phosphors` (specifying the emissions spectra for hte monitor spectra) and `Cones` (specifying the absorption spectra of human cones), and two (3×1) vectors specifying an RGB setting for the Phosphors. The function should return "1" if the two colors CAN be distinguished by a protanope, and "0" if two colors CANNOT be distinguished by a protanope. (2 points)

   Let $x_0 = (100; 100; 100)$ denote one setting for the monitor phosphors. Now test your function using x_0 and each of the following four RGB settings. Which color pairs are "ok", in the sense that your protanope friend will be able to distinguish x_0 from the other color? (1 point each).

   (*i*) $x_1 = (200; 100; 100)$

   (*ii*) $x_2 = (100; 150; 100)$

   (*iii*) $x_3 = (204; 76; 99)$

(*iv*) $x_4 = (7; 155; 97)$

b. Write an analogous function `isVisibleForDeuteranope.m` that serves the same function for people missing the M cone. Test the same four color pairs given above: x_0 vs. x_1, x_0 vs. x_2, etc.

c. Plot filled circles with each of the 5 colors given above. Matlab does not make this *super* easy but you can use the following command to plot the first one:

```
plot(0,1,'o', 'markersize', 50, 'markerfacecolor', [100 100 100]/255);
```

What this does is to tell matlab to plot an 'o' at the x, y location 0,1, make it big (marker size of 50), and fill it with color given by the RGB values [100 100 100] (where we divide by 255 because Matlab expects these to be numbers between 0 and 1: the vector [0 0 0] corresponds to black and [1 1 1] corresponds to white).

Now type `hold on;` to tell matlab to add the next plot command to the current axes without clearing them first. Then plot the second color using:

```
plot(1,1,'o', 'markersize', 50, 'markerfacecolor', [200 100 100]/255);
```

Complete the plot to show the other 3 colors above at locations (2,1), (3,1), and (4,1). Can you distinguish them? (I don't expect this to fool any dichromats unless someone is examining this on a well calibrated old-school CRT monitor!).

5. A clothing manufacturer, hoping to cash in on the recent furor over "the dress", decides to manufacture striped dresses that can be used to diagnose the three different forms of dichromacy, that is, the three forms of color blindness that result from loss of the L cone, M cone, or S cone. (I didn't say it was a *good* idea!)

   They have hired you – a neuroscientist with expertise in the linear algebra of color vision – to help them pick the pair of colors for each dress. Can you propose a pair of colors for three different dresses, one for Protanopia, another Deuteranopia, and a third for Tritanopia?

   (Let's assume for now that you just need to specify each color in terms of the 3-vector giving the intensities for the three monitor Phosphors used above; for now they just want a webpage showing their dresses).

   Make sure your color pairs are all physically realizable, i.e., they don't use values outside the range (0, 255), for each of the three phosphors.

   Make a plot showing the fabric for each dress: you can adapt the following command, which makes a plot with two colors of lines specified by the 3-vectors `col1` and `col2`

```
clf;  % clear plot
plot([0 1]', (0:.1:.3)'*[1 1], 'color', col1, 'linewidth', 50);
hold on;
plot([0 1]', (.05:.1:.35)'*[1 1], 'color', col2, 'linewidth', 50);
hold off;
```

   You can adjust the `linewidth` value if the lines look to fat or too thin. (3 points for each dress).

   Can you explain how you went about picking these colors in terms of null spaces?